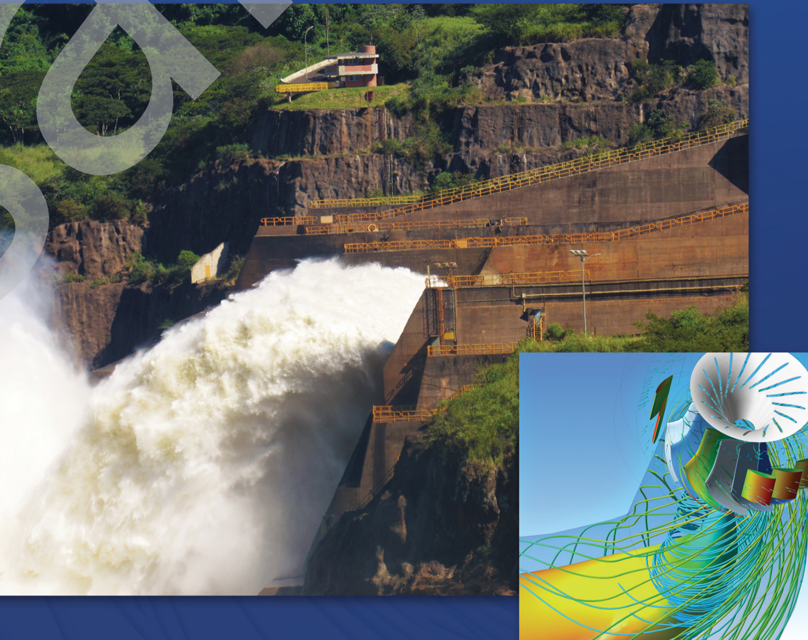
Fourth Edition

Fluid Mechanics

Fundamentals and Applications



Yunus A. Çengel
John M. Cimbala

Mc Graw Hill Education

FLUID MECHANICS

FUNDAMENTALS AND APPLICATIONS

FOURTH EDITION

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FLUID MECHANICS

FUNDAMENTALS AND APPLICATIONS

FOURTH EDITION

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FLUID MECHANICS: FUNDAMENTALS AND APPLICATIONS, FOURTH EDITION

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Dedication

To all students, with the hope of stimulating their desire to explore our marvelous world, of which fluid mechanics is a small but fascinating part. And to our wives Zehra and Suzy for their unending support.

About the Authors

Yunus A. Çengel is Professor Emeritus of Mechanical Engineering at the University of Nevada, Reno. He received his B.S. in mechanical engineering from Istanbul Technical University and his M.S. and Ph.D. in mechanical engineering from North Carolina State University. His research areas are renewable energy, desalination, exergy analysis, heat transfer enhancement, radiation heat transfer, and energy conservation. He served as the director of the Industrial Assessment Center (IAC) at the University of Nevada, Reno, from 1996 to 2000. He has led teams of engineering students to numerous manufacturing facilities in Northern Nevada and California to do industrial assessments, and has prepared energy conservation, waste minimization, and productivity enhancement reports for them.

Dr. Çengel is the coauthor of the widely adopted textbook *Thermodynamics: An Engineering Approach,* 8th edition (2015), published by McGraw-Hill Education. He is also the coauthor of the textbook *Heat and Mass Transfer: Fundamentals & Applications,* 5th Edition (2015), and the coauthor of the textbook *Fundamentals of Thermal-Fluid Sciences,* 5th edition (2017), both published by McGraw-Hill Education. Some of his textbooks have been translated to Chinese, Japanese, Korean, Spanish, Turkish, Italian, and Greek.

Dr. Çengel is the recipient of several outstanding teacher awards, and he has received the ASEE Meriam/Wiley Distinguished Author Award for excellence in authorship in 1992 and again in 2000.

Dr. Çengel is a registered Professional Engineer in the State of Nevada, and is a member of the American Society of Mechanical Engineers (ASME) and the American Society for Engineering Education (ASEE).

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Dr. Cimbala is the coauthor of three other textbooks: *Indoor Air Quality Engineering: Environmental Health and Control of Indoor Pollutants* (2003), published by Marcel-Dekker, Inc.; *Essentials of Fluid Mechanics: Fundamentals and Applications* (2008); and *Fundamentals of Thermal-Fluid Sciences,* 5th edition (2017), both published by McGraw-Hill Education. He has also contributed to parts of other books, and is the author or coauthor of dozens of journal and conference papers. He has also recently ventured into writing novels. More information can be found at www.mne.psu.edu/cimbala.

Professor Cimbala is the recipient of several outstanding teaching awards and views his book writing as an extension of his love of teaching. He is a member of the American Society of Mechanical Engineers (ASME), the American Society for Engineering Education (ASEE), and the American Physical Society (APS).

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Preface

BACKGROUND

Fluid mechanics is an exciting and fascinating subject with unlimited practical applications ranging from microscopic biological systems to automobiles, airplanes, and spacecraft propulsion. Fluid mechanics has also historically been one of the most challenging subjects for undergraduate students because proper analysis of fluid mechanics problems requires not only knowledge of the concepts but also physical intuition and experience. Our hope is that this book, through its careful explanations of concepts and its use of numerous practical examples, sketches, figures, and photographs, bridges the gap between knowledge and the proper application of that knowledge.

Fluid mechanics is a mature subject; the basic equations and approximations are well established and can be found in any introductory textbook. Our book is distinguished from other introductory books because we present the subject in a *progressive order* from simple to more difficult, building each chapter upon foundations laid down in earlier chapters. We provide more diagrams and photographs than other books because fluid mechanics is, by its nature, a highly visual subject. Only by illustrating the concepts discussed, can students fully appreciate the mathematical significance of the material.

OBJECTIVES

This book has been written for the first fluid mechanics course for undergraduate engineering students. There is sufficient material for a two-course sequence, if desired. We assume that readers will have an adequate background in calculus, physics, engineering mechanics, and thermodynamics. The objectives of this text are

- To present the *basic principles and equations* of fluid mechanics.
- To show numerous and diverse real-world *engineering examples* to give the student the intuition necessary for correct application of fluid mechanics principles in engineering applications.
- To develop an *intuitive understanding* of fluid mechanics by emphasizing the physics, and reinforcing that understanding through illustrative figures and photographs.

The book contains enough material to allow considerable flexibility in teaching the course. Aeronautics and aerospace engineers might emphasize potential flow, drag and lift, compressible flow, turbomachinery, and CFD, while mechanical or civil engineering instructors might choose to emphasize pipe flows and open-channel flows, respectively.

NEW TO THE FOURTH EDITION

All the popular features of the previous editions have been retained while new ones have been added. The main body of the text remains largely unchanged. A noticeable change is the addition of a number of exciting new pictures throughout the book.

Four new subsections have been added: "Uniform versus Nonuniform Flow" and "Equation Solvers" to Chap. 1, "Flying in Nature" by guest author Azar Eslam Panah of Penn State Berks to Chap. 11, and "CFD Methods for Two-Phase Flows" by guest author Alex Rattner of Penn State to Chap. 15. In Chap. 8, we now highlight the explicit Churchill equation as an alternative to the implicit Colebrook equation.

Two new Application Spotlights, have been added: "Smelling Food; the Human Airway" by Rui Ni of Penn State, to Chap. 4, and "Multicolor Particle Shadow Velocimetry/Accelerometry" by Michael McPhail and Michael Krane of Penn State to Chap. 8.

A large number of the end-of-chapter problems in the text have been modified and many problems were replaced by new ones. Also, several of the solved example problems have been replaced.

PHILOSOPHY AND GOAL

The Fourth Edition of *Fluid Mechanics: Fundamentals and Applications* has the same goals and philosophy as the other texts by lead author Yunus Çengel.

- Communicates directly with tomorrow's engineers in a *simple yet precise* manner
- Leads students toward a clear understanding and firm grasp of the *basic principles* of fluid mechanics
- Encourages creative thinking and development of a *deeper understanding* and *intuitive feel* for fluid mechanics
- Is read by students with *interest* and *enthusiasm* rather than merely as a guide to solve homework problems

The best way to learn is by practice. Special effort is made throughout the book to reinforce the material that was presented earlier (in each chapter as well as in material from previous chapters). Many of the illustrated example problems and end-of-chapter problems are comprehensive and encourage students to review and revisit concepts and intuitions gained previously.

Throughout the book, we show examples generated by computational fluid dynamics (CFD). We also provide an introductory chapter on the subject. Our goal is not to teach the details about numerical algorithms associated with CFD—this is more properly presented in a separate course. Rather, our intent is to introduce undergraduate students to the capabilities and limitations of CFD as an *engineering tool*. We use CFD solutions in much the same way as experimental results are used from wind tunnel tests (i.e., to reinforce understanding of the physics of fluid flows and to provide quality flow visualizations that help explain fluid behavior). With dozens of CFD end-of-chapter problems posted on the website, instructors have ample opportunity to introduce the basics of CFD throughout the course.

CONTENT AND ORGANIZATION

This book is organized into 15 chapters beginning with fundamental concepts of fluids, fluid properties, and fluid flows and ending with an introduction to computational fluid dynamics.

- Chapter 1 provides a basic introduction to fluids, classifications of fluid flow, control volume versus system formulations, dimensions, units, significant digits, and problem-solving techniques.

- Chapter 2 is devoted to fluid properties such as density, vapor pressure, specific heats, speed of sound, viscosity, and surface tension.
- Chapter 3 deals with fluid statics and pressure, including manometers and barometers, hydrostatic forces on submerged surfaces, buoyancy and stability, and fluids in rigid-body motion.
- Chapter 4 covers topics related to fluid kinematics, such as the differences between Lagrangian and Eulerian descriptions of fluid flows, flow patterns, flow visualization, vorticity and rotationality, and the Reynolds transport theorem.
- Chapter 5 introduces the fundamental conservation laws of mass, momentum, and energy, with emphasis on the proper use of the mass, Bernoulli, and energy equations and the engineering applications of these equations.
- Chapter 6 applies the Reynolds transport theorem to linear momentum and angular momentum and emphasizes practical engineering applications of finite control volume momentum analysis.
- Chapter 7 reinforces the concept of dimensional homogeneity and introduces the Buckingham Pi theorem of dimensional analysis, dynamic similarity, and the method of repeating variables—material that is useful throughout the rest of the book and in many disciplines in science and engineering.
- Chapter 8 is devoted to flow in pipes and ducts. We discuss the differences between laminar and turbulent flow, friction losses in pipes and ducts, and minor losses in piping networks. We also explain how to properly select a pump or fan to match a piping network. Finally, we discuss various experimental devices that are used to measure flow rate and velocity, and provide a brief introduction to biofluid mechanics.
- Chapter 9 deals with differential analysis of fluid flow and includes derivation and application of the continuity equation, the Cauchy equation, and the Navier–Stokes equation. We also introduce the stream function and describe its usefulness in analysis of fluid flows, and we provide a brief introduction to biofluids. Finally, we point out some of the unique aspects of differential analysis related to biofluid mechanics.
- Chapter 10 discusses several *approximations* of the Navier–Stokes equation and provides example solutions for each approximation, including creeping flow, inviscid flow, irrotational (potential) flow, and boundary layers.
- Chapter 11 covers forces on living and non-living bodies (drag and lift), explaining the distinction between friction and pressure drag, and providing drag coefficients for many common geometries. This chapter emphasizes the practical application of wind tunnel measurements coupled with dynamic similarity and dimensional analysis concepts introduced earlier in Chap. 7.
- Chapter 12 extends fluid flow analysis to compressible flow, where the behavior of gases is greatly affected by the Mach number. In this chapter, the concepts of expansion waves, normal and oblique shock waves, and choked flow are introduced.
- Chapter 13 deals with open-channel flow and some of the unique features associated with the flow of liquids with a free surface, such as surface waves and hydraulic jumps.

- Chapter 14 examines turbomachinery in more detail, including pumps, fans, and turbines. An emphasis is placed on how pumps and turbines work, rather than on their detailed design. We also discuss overall pump and turbine design, based on dynamic similarity laws and simplified velocity vector analyses.
- Chapter 15 describes the fundamental concepts of computational fluid dyamics (CFD) and shows students how to use commercial CFD codes as tools to solve complex fluid mechanics problems. We emphasize the *application* of CFD rather than the algorithms used in CFD codes.

Each chapter contains a wealth of end-of-chapter homework problems. Most of the problems that require calculation use the SI system of units; however, about 20 percent use English units. A comprehensive set of appendices is provided, giving the thermodynamic and fluid properties of several materials, in addition to air and water, along with some useful plots and tables. Many of the end-of-chapter problems require the use of material properties from the appendices to enhance the realism of the problems.

LEARNING TOOLS

EMPHASIS ON PHYSICS

A distinctive feature of this book is its emphasis on the physical aspects of the subject matter in addition to mathematical representations and manipulations. The authors believe that the emphasis in undergraduate education should remain on *developing a sense of underlying physical mechanisms* and a *mastery of solving practical problems* that an engineer is likely to face in the real world. Developing an intuitive understanding should also make the course a more motivating and worthwhile experience for the students.

EFFECTIVE USE OF ASSOCIATION

An observant mind should have no difficulty understanding engineering sciences. After all, the principles of engineering sciences are based on our *everyday experiences* and *experimental observations*. Therefore, a physical, intuitive approach is used throughout this text. Frequently, *parallels are drawn* between the subject matter and students' everyday experiences so that they can relate the subject matter to what they already know.

SELF-INSTRUCTING

The material in the text is introduced at a level that an average student can follow comfortably. It speaks *to* students, not *over* students. In fact, it is *self-instructive*. Noting that the principles of science are based on experimental observations, most of the derivations in this text are largely based on physical arguments, and thus they are easy to follow and understand.

EXTENSIVE USE OF ARTWORK AND PHOTOGRAPHS

Figures are important learning tools that help the students "get the picture," and the text makes effective use of graphics. It contains more figures, photographs, and illustrations than any other book in this category. Figures attract attention and stimulate curiosity and interest. Most of the figures in this text are intended to serve as a means of emphasizing some key concepts that would otherwise go unnoticed; some serve as page summaries.

CONSISTENT COLOR SCHEME FOR FIGURES

The figures have a consistent color scheme applied for all arrows.

- Blue: (⟶) motion related, like velocity vectors
- Green: (⟶) force and pressure related, and torque
- Black: (⟶) distance related arrows and dimensions
- Red: (⟶) energy related, like heat and work
- Purple: (⟶) acceleration and gravity vectors, vorticity, and miscellaneous

NUMEROUS WORKED-OUT EXAMPLES

All chapters contain numerous worked-out *examples* that both clarify the material and illustrate the use of basic principles in a context that helps develop the student's intuition. An *intuitive* and *systematic* approach is used in the solution of all example problems. The solution methodology starts with a statement of the problem, and all objectives are identified. The assumptions and approximations are then stated together with their justifications. Any properties needed to solve the problem are listed separately. Numerical values are used together with numbers to emphasize that without units, numbers are meaningless. The significance of each example's result is discussed following the solution. This methodical approach is also followed and provided in the solutions to the end-of-chapter problems, available to instructors.

A WEALTH OF REALISTIC END-OF-CHAPTER PROBLEMS

The end-of-chapter problems are grouped under specific topics to make problem selection easier for both instructors and students. Within each group of problems are *Concept Questions,* indicated by "C," to check the students' level of understanding of basic concepts. Problems under *Fundamentals of Engineering (FE) Exam Problems* are designed to help students prepare for the *Fundamentals of Engineering* exam, as they prepare for their Professional Engineering license. The problems under *Review Problems* are more comprehensive in nature and are not directly tied to any specific section of a chapter—in some cases they require review of material learned in previous chapters. Problems designated as *Design and Essay* are intended to encourage students to make engineering judgments, to conduct independent exploration of topics of interest, and to communicate their findings in a professional manner. Problems designated by an "E" are in English units, and SI users can ignore them. Problems with the icon are comprehensive in nature and are intended to be solved with a computer, using appropriate software. Several economics- and safety-related problems are incorporated throughout to enhance cost and safety awareness among engineering students. Answers to selected problems are listed immediately following the problem for convenience to students.

USE OF COMMON NOTATION

The use of different notation for the same quantities in different engineering courses has long been a source of discontent and confusion. A student taking both fluid mechanics and heat transfer, for example, has to use the notation Q for volume flow rate in one course, and for heat transfer in the other. The need to unify notation in engineering education has often been raised, even in some reports of conferences sponsored by the National Science Foundation through

Foundation Coalitions, but little effort has been made to date in this regard. For example, refer to the final report of the *Mini-Conference on Energy Stem Innovations,* May 28 and 29, 2003, University of Wisconsin. In this text we made a conscious effort to minimize this conflict by adopting the familiar thermodynamic notation $\dot{V}$ for volume flow rate, thus reserving the notation Q for heat transfer. Also, we consistently use an overdot to denote time rate. We think that both students and instructors will appreciate this effort to promote a common notation.

A CHOICE OF SI ALONE OR SI/ENGLISH UNITS

In recognition of the fact that English units are still widely used in some industries, both SI and English units are used in this text, with an emphasis on SI. The material in this text can be covered using combined SI/English units or SI units alone, depending on the preference of the instructor. The property tables and charts in the appendices are presented in both units, except the ones that involve dimensionless quantities. Problems, tables, and charts in English units are designated by "E" after the number for easy recognition, and they can be ignored easily by the SI users.

COMBINED COVERAGE OF BERNOULLI AND ENERGY EQUATIONS

The Bernoulli equation is one of the most frequently used equations in fluid mechanics, but it is also one of the most misused. Therefore, it is important to emphasize the limitations on the use of this idealized equation and to show how to properly account for imperfections and irreversible losses. In Chap. 5, we do this by introducing the energy equation right after the Bernoulli equation and demonstrating how the solutions of many practical engineering problems differ from those obtained using the Bernoulli equation. This helps students develop a realistic view of the Bernoulli equation.

A SEPARATE CHAPTER ON CFD

Commercial *Computational Fluid Dynamics* (CFD) codes are widely used in engineering practice in the design and analysis of flow systems, and it has become exceedingly important for engineers to have a solid understanding of the fundamental aspects, capabilities, and limitations of CFD. Recognizing that most undergraduate engineering curriculums do not have room for a full course on CFD, a separate chapter is included here to make up for this deficiency and to equip students with an adequate background on the strengths and weaknesses of CFD.

APPLICATION SPOTLIGHTS

Throughout the book are highlighted examples called *Application Spotlights* where a real-world application of fluid mechanics is shown. A unique feature of these special examples is that they are written by *guest authors*. The Application Spotlights are designed to show students how fluid mechanics has diverse applications in a wide variety of fields. They also include eye-catching photographs from the guest authors' research.

GLOSSARY OF FLUID MECHANICS TERMS

Throughout the chapters, when an important key term or concept is introduced and defined, it appears in **black** boldface type. Fundamental fluid mechanics terms and concepts appear in **red** boldface type, and these fundamental terms also appear in a comprehensive end-of-book glossary developed by Professor Emeritus James Brasseur of The Pennsylvania State University. This unique glossary is an excellent learning and review tool for students as they move forward in their study of fluid mechanics.

CONVERSION FACTORS

Frequently used conversion factors, physical constants, and properties of air and water at 20°C and atmospheric pressure are listed at the very end of the book for easy reference.

NOMENCLATURE

A list of the major symbols, subscripts, and superscripts used in the text is provided near the end of the book for easy reference.

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Finally, special thanks must go to our families, especially our wives, Zehra Çengel and Suzanne Cimbala, for their continued patience, understanding, and support throughout the preparation of this book, which involved many long hours when they had to handle family concerns on their own because their husbands' faces were glued to a computer screen.

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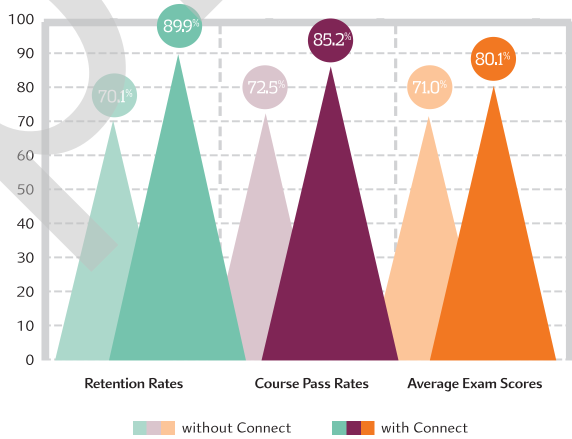

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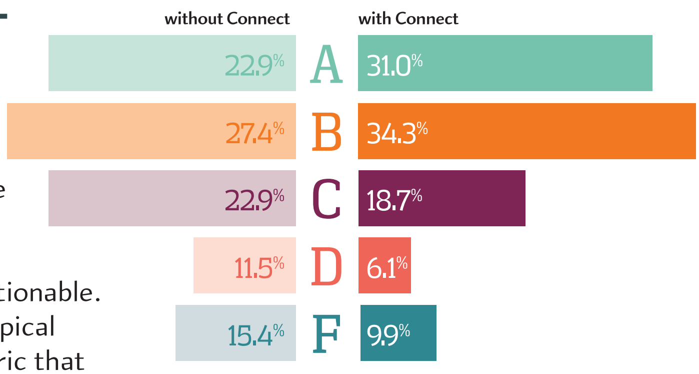

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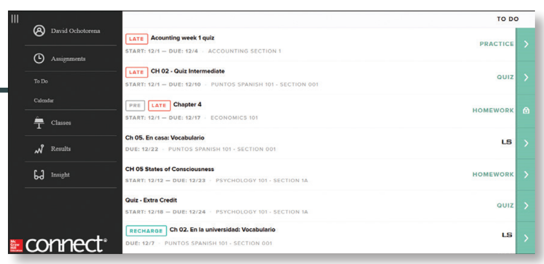

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Online Resources for Instructors

Online Resources available at www.mhhe.com/cengel

Your home page for teaching fluid mechanics, the *Fluid Mechanics: Fundamentals and Applications* text-specific website is password protected and offers resources for instructors.

- **Electronic Solutions Manual—**provides PDF files with detailed typed solutions to all text homework problems.
- **Lecture Slides**—provide PowerPoint lecture slides for all chapters.
- **Fluid Mechanics Videos**—provide visual illustrations and demonstrations of fluid mechanics concepts for deeper student comprehension.
- **Image Library**—provides electronic files for text figures for easy integration into your course presentations, exams, and assignments.
- **Sample Syllabi**—make it easier for you to map out your course using this text for different course durations (one quarter, one semester, etc.) and for different disciplines (ME approach, Civil approach, etc.).
- **Transition Guides**—compare coverage to other popular introductory fluid mechanics books at the section level to aid transition to teaching from our text.
- CFD homework problems and solutions designed for use with various CFD packages.
- **COSMOS**—allows instructors to create custom homework, quizzes, and tests using end-of-chapter problems and correlating solutions.

FLUID MECHANICS

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CHAPTER

1

INTRODUCTION AND BASIC CONCEPTS

In this introductory chapter, we present the basic concepts commonly used in the analysis of fluid flow. We start this chapter with a discussion of the phases of matter and the numerous ways of classification of fluid flow, such as *viscous versus inviscid regions of flow, internal versus external flow, compressible versus incompressible flow, laminar versus turbulent flow, natural versus forced flow,* and *steady versus unsteady flow*. We also discuss the *no-slip condition* at solid–fluid interfaces and present a brief history of the development of fluid mechanics.

After presenting the concepts of *system* and *control volume,* we review the *unit systems* that will be used. We then discuss how mathematical models for engineering problems are prepared and how to interpret the results obtained from the analysis of such models. This is followed by a presentation of an intuitive systematic *problem-solving technique* that can be used as a model in solving engineering problems. Finally, we discuss accuracy, precision, and significant digits in engineering measurements and calculations.

OBJECTIVES

When you finish reading this chapter, you should be able to

- Understand the basic concepts of fluid mechanics
- Recognize the various types of fluid flow problems encountered in practice
- Model engineering problems and solve them in a systematic manner
- Have a working knowledge of accuracy, precision, and significant digits, and recognize the importance of dimensional homogeneity in engineering calculations

Schlieren image showing the thermal plume produced by Professor Cimbala as he welcomes you to the fascinating world of fluid mechanics.
Courtesy of Michael J. Hargather and John Cimbala.

FIGURE 1–1
Fluid mechanics deals with liquids and gases in motion or at rest.
© Goodshoot/Fotosearch RF

1–1 ▪ INTRODUCTION

Mechanics is the oldest physical science that deals with both stationary and moving bodies under the influence of forces. The branch of mechanics that deals with bodies at rest is called **statics**, while the branch that deals with bodies in motion under the action of forces is called **dynamics**. The subcategory **fluid mechanics** is defined as the science that deals with the behavior of fluids at rest (*fluid statics*) or in motion (*fluid dynamics*), and the interaction of fluids with solids or other fluids at the boundaries. Fluid mechanics is also referred to as **fluid dynamics** by considering fluids at rest as a special case of motion with zero velocity (Fig. 1–1).

Fluid mechanics itself is also divided into several categories. The study of the motion of fluids that can be approximated as incompressible (such as liquids, especially water, and gases at low speeds) is usually referred to as **hydrodynamics**. A subcategory of hydrodynamics is **hydraulics**, which deals with liquid flows in pipes and open channels. **Gas dynamics** deals with the flow of fluids that undergo significant density changes, such as the flow of gases through nozzles at high speeds. The category **aerodynamics** deals with the flow of gases (especially air) over bodies such as aircraft, rockets, and automobiles at high or low speeds. Some other specialized categories such as **meteorology**, **oceanography**, and **hydrology** deal with naturally occurring flows.

What Is a Fluid?

You will recall from physics that a substance exists in three primary phases: solid, liquid, and gas. (At very high temperatures, it also exists as plasma.) A substance in the liquid or gas phase is referred to as a **fluid**. Distinction between a solid and a fluid is made on the basis of the substance's ability to resist an applied shear (or tangential) stress that tends to change its shape. A solid can resist an applied shear stress by deforming, whereas *a fluid deforms continuously under the influence of a shear stress,* no matter how small. In solids, stress is proportional to *strain,* but in fluids, stress is proportional to *strain rate*. When a constant shear force is applied, a solid eventually stops deforming at some fixed strain angle, whereas a fluid never stops deforming and approaches a constant *rate* of strain.

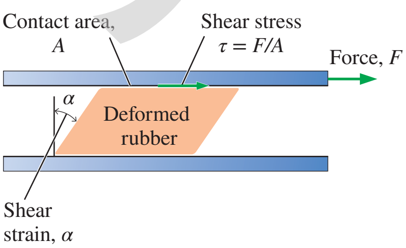


FIGURE 1–2
Deformation of a rubber block placed between two parallel plates under the influence of a shear force. The shear stress shown is that on the rubber—an equal but opposite shear stress acts on the upper plate.

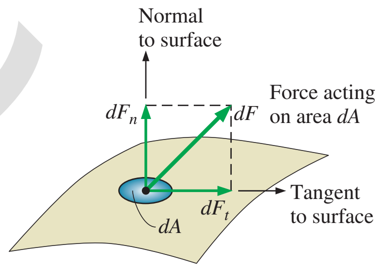
Consider a rectangular rubber block tightly placed between two plates. As the upper plate is pulled with a force F while the lower plate is held fixed, the rubber block deforms, as shown in Fig. 1–2. The angle of deformation α (called the *shear strain* or *angular displacement*) increases in proportion to the applied force F. Assuming there is no slip between the rubber and the plates, the upper surface of the rubber is displaced by an amount equal to the displacement of the upper plate while the lower surface remains stationary. In equilibrium, the net force acting on the upper plate in the horizontal direction must be zero, and thus a force equal and opposite to F must be acting on the plate. This opposing force that develops at the plate–rubber interface due to friction is expressed as $F = \tau A$, where τ is the shear stress and A is the contact area between the upper plate and the rubber. When the force is removed, the rubber returns to its original position. This phenomenon would also be observed with other solids such as a steel block provided that the applied force does not exceed the elastic range. If this experiment were repeated with a fluid (with two large parallel plates placed in a large body of water, for example), the fluid layer in contact with the upper plate would move with the plate continuously at the velocity of the plate no matter how small the force *F*. The fluid velocity would decrease with depth because of friction between fluid layers, reaching zero at the lower plate.

You will recall from statics that **stress** is defined as force per unit area and is determined by dividing the force by the area upon which it acts. The normal component of a force acting on a surface per unit area is called the **normal stress**, and the tangential component of a force acting on a surface per unit area is called **shear stress** (Fig. 1–3). In a fluid at rest, the normal stress is called **pressure**. A fluid at rest is at a state of zero shear stress. When the walls are removed or a liquid container is tilted, a shear develops as the liquid moves to re-establish a horizontal free surface.

In a liquid, groups of molecules can move relative to each other, but the volume remains relatively constant because of the strong cohesive forces between the molecules. As a result, a liquid takes the shape of the container it is in, and it forms a free surface in a larger container in a gravitational field. A gas, on the other hand, expands until it encounters the walls of the container and fills the entire available space. This is because the gas molecules are widely spaced, and the cohesive forces between them are very small. Unlike liquids, a gas in an open container cannot form a free surface (Fig. 1–4).

Although solids and fluids are easily distinguished in most cases, this distinction is not so clear in some borderline cases. For example, *asphalt* appears and behaves as a solid since it resists shear stress for short periods of time. When these forces are exerted over extended periods of time, however, the asphalt deforms slowly, behaving as a fluid. Some plastics, lead, and slurry mixtures exhibit similar behavior. Such borderline cases are beyond the scope of this text. The fluids we deal with in this text will be clearly recognizable as fluids.

Intermolecular bonds are strongest in solids and weakest in gases. One reason is that molecules in solids are closely packed together, whereas in gases they are separated by relatively large distances (Fig. 1–5). The molecules in a solid are arranged in a pattern that is repeated throughout. Because of the small distances between molecules in a solid, the attractive forces of molecules on each other are large and keep the molecules at fixed positions. The molecular spacing in the liquid phase is not much different from that of



Normal stress: $\sigma = \dfrac{dF_n}{dA}$

Shear stress: $\tau = \dfrac{dF_t}{dA}$

FIGURE 1–3
The normal stress and shear stress at the surface of a fluid element. For fluids at rest, the shear stress is zero and pressure is the only normal stress.

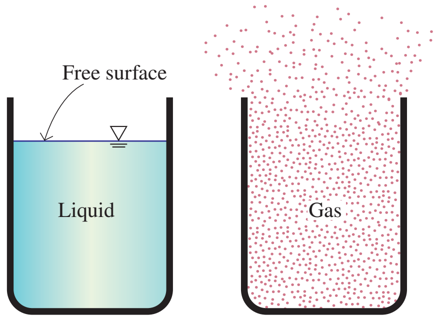
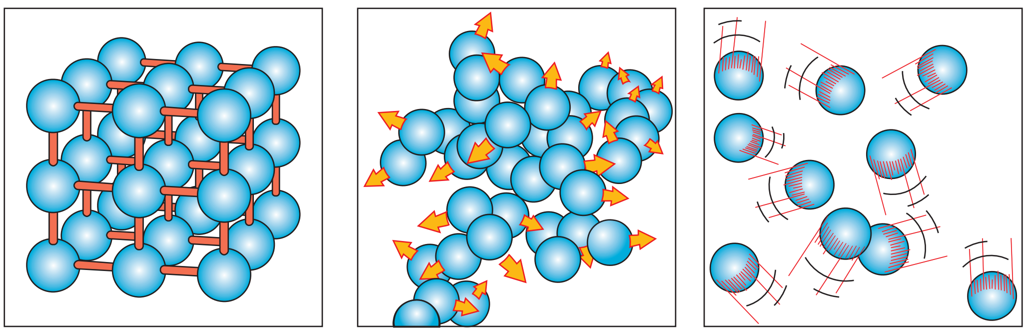


FIGURE 1–4
Unlike a liquid, a gas does not form a free surface, and it expands to fill the entire available space.



(*a*) (*b*) (*c*)

FIGURE 1–5
The arrangement of atoms in different phases: (*a*) molecules are at relatively fixed positions in a solid, (*b*) groups of molecules move about each other in the liquid phase, and (*c*) individual molecules move about at random in the gas phase.

the solid phase, except the molecules are no longer at fixed positions relative to each other and they can rotate and translate freely. In a liquid, the intermolecular forces are weaker relative to solids, but still strong compared with gases. The distances between molecules generally increase slightly as a solid turns liquid, with water being a notable exception.

In the gas phase, the molecules are far apart from each other, and molecular ordering is nonexistent. Gas molecules move about at random, continually colliding with each other and the walls of the container in which they are confined. Particularly at low densities, the intermolecular forces are very small, and collisions are the only mode of interaction between the molecules. Molecules in the gas phase are at a considerably higher energy level than they are in the liquid or solid phase. Therefore, the gas must release a large amount of its energy before it can condense or freeze.

Gas and *vapor* are often used as synonymous words. The vapor phase of a substance is customarily called a *gas* when it is above the critical temperature. *Vapor* usually implies that the current phase is not far from a state of condensation.

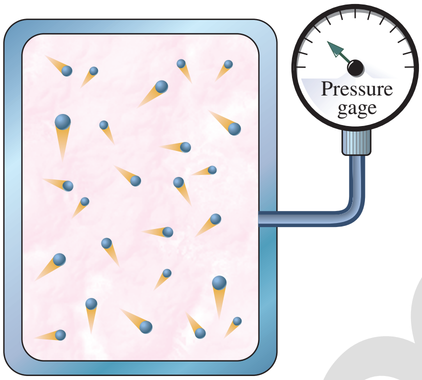


FIGURE 1–6
On a microscopic scale, pressure is determined by the interaction of individual gas molecules. However, we can measure the pressure on a macroscopic scale with a pressure gage.

Any practical fluid system consists of a large number of molecules, and the properties of the system naturally depend on the behavior of these molecules. For example, the pressure of a gas in a container is the result of momentum transfer between the molecules and the walls of the container. However, one does not need to know the behavior of the gas molecules to determine the pressure in the container. It is sufficient to attach a pressure gage to the container (Fig. 1–6). This macroscopic or *classical* approach does not require a knowledge of the behavior of individual molecules and provides a direct and easy way to analyze engineering problems. The more elaborate microscopic or *statistical* approach, based on the average behavior of large groups of individual molecules, is rather involved and is used in this text only in a supporting role.

Application Areas of Fluid Mechanics

It is important to develop a good understanding of the basic principles of fluid mechanics, since fluid mechanics is widely used both in everyday activities and in the design of modern engineering systems from vacuum cleaners to supersonic aircraft. For example, fluid mechanics plays a vital role in the human body. The heart is constantly pumping blood to all parts of the human body through the arteries and veins, and the lungs are the sites of airflow in alternating directions. All artificial hearts, breathing machines, and dialysis systems are designed using fluid dynamics (Fig. 1–7).

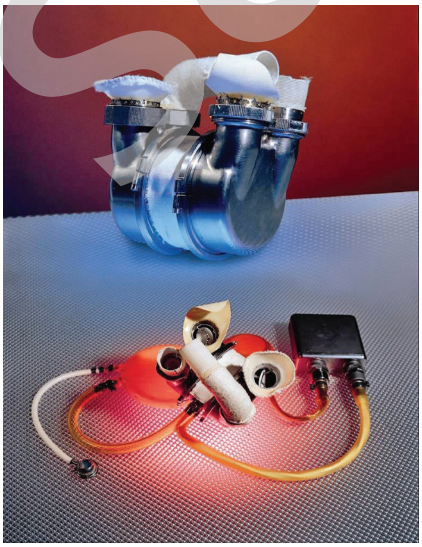
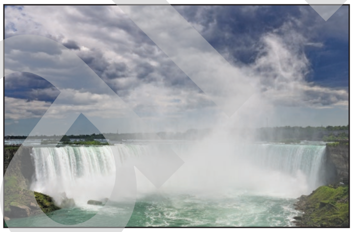


FIGURE 1–7
Fluid dynamics is used extensively in the design of artificial hearts. Shown here is the Penn State Electric Total Artificial Heart.
Courtesy of the Biomedical Photography Lab, Penn State Biomedical Engineering Institute. Used by permission.

An ordinary house is, in some respects, an exhibition hall filled with applications of fluid mechanics. The piping systems for water, natural gas, and sewage for an individual house and the entire city are designed primarily on the basis of fluid mechanics. The same is also true for the piping and ducting network of heating and air-conditioning systems. A refrigerator involves tubes through which the refrigerant flows, a compressor that pressurizes the refrigerant, and two heat exchangers where the refrigerant absorbs and rejects heat. Fluid mechanics plays a major role in the design of all these components. Even the operation of ordinary faucets is based on fluid mechanics.

We can also see numerous applications of fluid mechanics in an automobile. All components associated with the transportation of the fuel from the fuel tank to the cylinders—the fuel line, fuel pump, and fuel injectors or carburetors—as well as the mixing of the fuel and the air in the cylinders and the purging of combustion gases in exhaust pipes—are analyzed using fluid mechanics. Fluid mechanics is also used in the design of the heating and air-conditioning system, the hydraulic brakes, the power steering, the automatic transmission, the lubrication systems, the cooling system of the engine block including the radiator and the water pump, and even the tires. The sleek streamlined shape of recent model cars is the result of efforts to minimize drag by using extensive analysis of flow over surfaces.

On a broader scale, fluid mechanics plays a major part in the design and analysis of aircraft, boats, submarines, rockets, jet engines, wind turbines, biomedical devices, cooling systems for electronic components, and transportation systems for moving water, crude oil, and natural gas. It is also considered in the design of buildings, bridges, and even billboards to make sure that the structures can withstand wind loading. Numerous natural phenomena such as the rain cycle, weather patterns, the rise of ground water to the tops of trees, winds, ocean waves, and currents in large water bodies are also governed by the principles of fluid mechanics (Fig. 1–8).



Natural flows and weather
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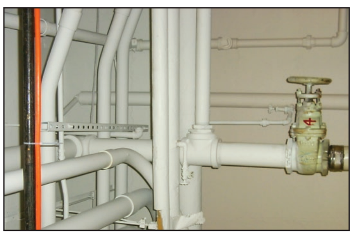

Piping and plumbing systems
Photo by John M. Cimbala

Industrial applications
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FIGURE 1–8
Some application areas of fluid mechanics.

FIGURE 1–9
Segment of Pergamon pipeline. Each clay pipe section was 13 to 18 cm in diameter.
Courtesy of Gunther Garbrecht. Used by permission.

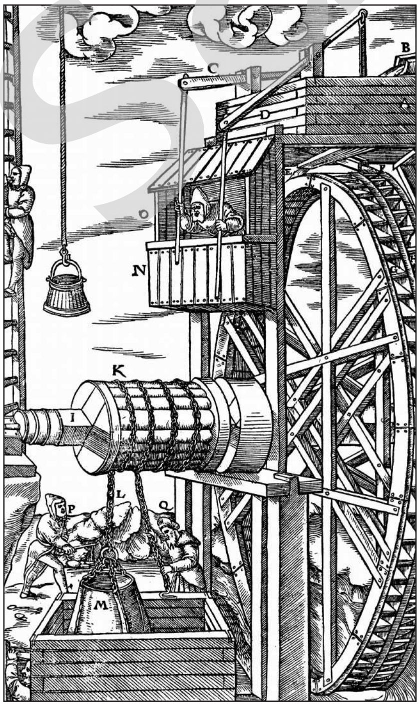

FIGURE 1–10
A mine hoist powered by a reversible water wheel.
© Universal History Archive/Getty Images

1–2 ▪ A BRIEF HISTORY OF FLUID MECHANICS[1]

One of the first engineering problems humankind faced as cities were developed was the supply of water for domestic use and irrigation of crops. Our urban lifestyles can be retained only with abundant water, and it is clear from archeology that every successful civilization of prehistory invested in the construction and maintenance of water systems. The Roman aqueducts, some of which are still in use, are the best known examples. However, perhaps the most impressive engineering from a technical viewpoint was done at the Hellenistic city of Pergamon in present-day Turkey. There, from 283 to 133 BC, they built a series of pressurized lead and clay pipelines (Fig. 1–9), up to 45 km long that operated at pressures exceeding 1.7 MPa (180 m of head). Unfortunately, the names of almost all these early builders are lost to history.

The earliest recognized contribution to fluid mechanics theory was made by the Greek mathematician Archimedes (285–212 BC). He formulated and applied the buoyancy principle in history's first nondestructive test to determine the gold content of the crown of King Hiero II. The Romans built great aqueducts and educated many conquered people on the benefits of clean water, but overall had a poor understanding of fluids theory. (Perhaps they shouldn't have killed Archimedes when they sacked Syracuse.)

During the Middle Ages, the application of fluid machinery slowly but steadily expanded. Elegant piston pumps were developed for dewatering mines, and the watermill and windmill were perfected to grind grain, forge metal, and for other tasks. For the first time in recorded human history, significant work was being done without the power of a muscle supplied by a person or animal, and these inventions are generally credited with enabling the later industrial revolution. Again the creators of most of the progress are unknown, but the devices themselves were well documented by several technical writers such as Georgius Agricola (Fig. 1–10).

The Renaissance brought continued development of fluid systems and machines, but more importantly, the scientific method was perfected and adopted throughout Europe. Simon Stevin (1548–1617), Galileo Galilei (1564–1642), Edme Mariotte (1620–1684), and Evangelista Torricelli (1608–1647) were among the first to apply the method to fluids as they investigated hydrostatic pressure distributions and vacuums. That work was integrated and refined by the brilliant mathematician and philosopher, Blaise Pascal (1623–1662). The Italian monk, Benedetto Castelli (1577–1644) was the first person to publish a statement of the continuity principle for fluids. Besides formulating his equations of motion for solids, Sir Isaac Newton (1643–1727) applied his laws to fluids and explored fluid inertia and resistance, free jets, and viscosity. That effort was built upon by Daniel Bernoulli (1700–1782), a Swiss, and his associate Leonard Euler (1707–1783). Together, their work defined the energy and momentum equations. Bernoulli's 1738 classic treatise *Hydrodynamica* may be considered the first fluid mechanics text. Finally, Jean d'Alembert (1717–1789) developed the idea of velocity and acceleration components, a differential expression of

[1] This section is contributed by Professor Glenn Brown of Oklahoma State University.

continuity, and his "paradox" of zero resistance to steady uniform motion over a body.

The development of fluid mechanics theory through the end of the eighteenth century had little impact on engineering since fluid properties and parameters were poorly quantified, and most theories were abstractions that could not be quantified for design purposes. That was to change with the development of the French school of engineering led by Riche de Prony (1755–1839). Prony (still known for his brake to measure shaft power) and his associates in Paris at the École Polytechnique and the École des Ponts et Chaussées were the first to integrate calculus and scientific theory into the engineering curriculum, which became the model for the rest of the world. (So now you know whom to blame for your painful freshman year.) Antonie Chezy (1718–1798), Louis Navier (1785–1836), Gaspard Coriolis (1792–1843), Henry Darcy (1803–1858), and many other contributors to fluid engineering and theory were students and/or instructors at the schools.

By the mid nineteenth century, fundamental advances were coming on several fronts. The physician Jean Poiseuille (1799–1869) had accurately measured flow in capillary tubes for multiple fluids, while in Germany Gotthilf Hagen (1797–1884) had differentiated between laminar and turbulent flow in pipes. In England, Lord Osborne Reynolds (1842–1912) continued that work (Fig. 1–11) and developed the dimensionless number that bears his name. Similarly, in parallel to the early work of Navier, George Stokes (1819–1903) completed the general equation of fluid motion (with friction) that takes their names. William Froude (1810–1879) almost singlehandedly developed the procedures and proved the value of physical model testing. American expertise had become equal to the Europeans as demonstrated by James Francis' (1815–1892) and Lester Pelton's (1829–1908) pioneering work in turbines and Clemens Herschel's (1842–1930) invention of the Venturi meter.

In addition to Reynolds and Stokes, many notable contributions were made to fluid theory in the late nineteenth century by Irish and English scientists, including William Thomson, Lord Kelvin (1824–1907), William Strutt, Lord Rayleigh (1842–1919), and Sir Horace Lamb (1849–1934). These individuals investigated a large number of problems, including dimensional analysis, irrotational flow, vortex motion, cavitation, and waves. In a broader sense,

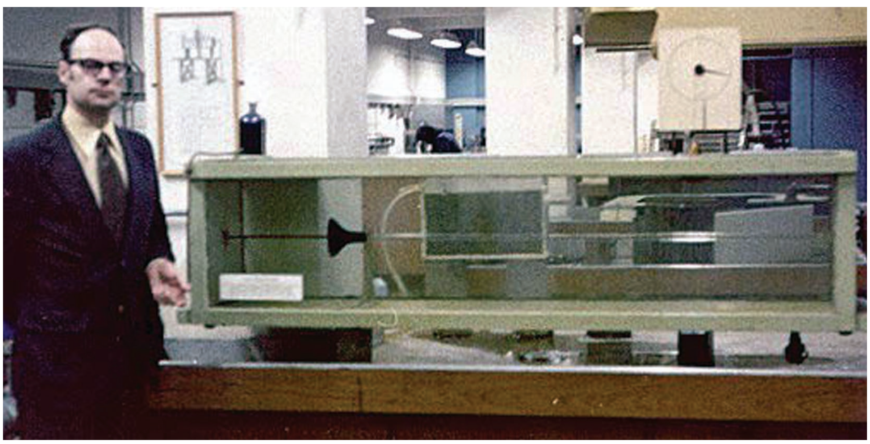

FIGURE 1–11
Osborne Reynolds' original apparatus for demonstrating the onset of turbulence in pipes, being operated by John Lienhard at the University of Manchester in 1975.

Courtesy of John Lienhard, University of Houston. Used by Permission.

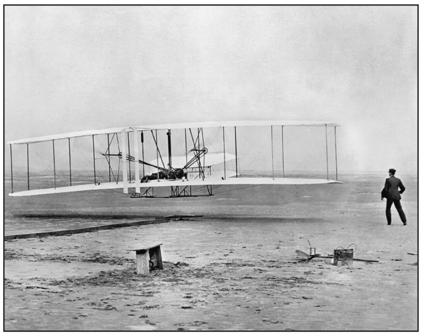

FIGURE 1–12
The Wright brothers take flight at Kitty Hawk.
Courtesy Library of Congress Prints & Photographs Division [LC-DIG-ppprs-00626].

FIGURE 1–13
Old and new wind turbine technologies north of Woodward, OK. The modern turbines have up to 8 MW capacities.
Photo courtesy of the Oklahoma Wind Power Initiative. Used by permission.

their work also explored the links between fluid mechanics, thermodynamics, and heat transfer.

The dawn of the twentieth century brought two monumental developments. First, in 1903, the self-taught Wright brothers (Wilbur, 1867–1912; Orville, 1871–1948) invented the airplane through application of theory and determined experimentation. Their primitive invention was complete and contained all the major aspects of modern aircraft (Fig. 1–12). The Navier–Stokes equations were of little use up to this time because they were too difficult to solve. In a pioneering paper in 1904, the German Ludwig Prandtl (1875–1953) showed that fluid flows can be divided into a layer near the walls, the *boundary layer,* where the friction effects are significant, and an outer layer where such effects are negligible and the simplified Euler and Bernoulli equations are applicable. His students, Theodor von Kármán (1881–1963), Paul Blasius (1883–1970), Johann Nikuradse (1894–1979), and others, built on that theory in both hydraulic and aerodynamic applications. (During World War II, both sides benefited from the theory as Prandtl remained in Germany while his best student, the Hungarian-born von Kármán, worked in America.)

The mid twentieth century could be considered a golden age of fluid mechanics applications. Existing theories were adequate for the tasks at hand, and fluid properties and parameters were well defined. These supported a huge expansion of the aeronautical, chemical, industrial, and water resources sectors; each of which pushed fluid mechanics in new directions. Fluid mechanics research and work in the late twentieth century were dominated by the development of the digital computer in America. The ability to solve large complex problems, such as global climate modeling or the optimization of a turbine blade, has provided a benefit to our society that the eighteenth-century developers of fluid mechanics could never have imagined (Fig. 1–13). The principles presented in the following pages have been applied to flows ranging from a moment at the microscopic scale to 50 years of simulation for an entire river basin. It is truly mind-boggling.

Where will fluid mechanics go in the twenty-first century and beyond? Frankly, even a limited extrapolation beyond the present would be sheer folly. However, if history tells us anything, it is that engineers will be applying what they know to benefit society, researching what they don't know, and having a great time in the process.

1–3 ▪ THE NO-SLIP CONDITION

Fluid flow is often confined by solid surfaces, and it is important to understand how the presence of solid surfaces affects fluid flow. We know that water in a river cannot flow through large rocks, and must go around them. That is, the water velocity normal to the rock surface must be zero, and water approaching the surface normally comes to a complete stop at the surface. What is not as obvious is that water approaching the rock at any angle also comes to a complete stop at the rock surface, and thus the tangential velocity of water at the surface is also zero.

Consider the flow of a fluid in a stationary pipe or over a solid surface that is nonporous (i.e., impermeable to the fluid). All experimental observations indicate that a fluid in motion comes to a complete stop at the surface and assumes a zero velocity relative to the surface. That is, a fluid in direct contact with a solid "sticks" to the surface, and there is no slip. This is known as the **no-slip condition**. The fluid property responsible for the no-slip condition and the development of the boundary layer is *viscosity* and is discussed in Chap. 2.

The photograph in Fig. 1–14 clearly shows the evolution of a velocity gradient as a result of the fluid sticking to the surface of a blunt nose. The layer that sticks to the surface slows the adjacent fluid layer because of viscous forces between the fluid layers, which slows the next layer, and so on. A consequence of the no-slip condition is that all velocity profiles must have zero values with respect to the surface at the points of contact between a fluid and a solid surface (Fig. 1–15). Therefore, the no-slip condition is responsible for the development of the velocity profile. The flow region adjacent to the wall in which the viscous effects (and thus the velocity gradients) are significant is called the **boundary layer**. Another consequence of the no-slip condition is the *surface drag,* or *skin friction drag,* which is the force a fluid exerts on a surface in the flow direction.

When a fluid is forced to flow over a curved surface, such as the back side of a cylinder, the boundary layer may no longer remain attached to the surface and separates from the surface—a process called **flow separation** (Fig. 1–16). We emphasize that the no-slip condition applies *everywhere* along the surface, even downstream of the separation point. Flow separation is discussed in greater detail in Chap. 9.

A phenomenon similar to the no-slip condition occurs in heat transfer. When two bodies at different temperatures are brought into contact, heat transfer occurs such that both bodies assume the same temperature at the points of contact. Therefore, a fluid and a solid surface have the same temperature at the points of contact. This is known as **no-temperature-jump condition**.

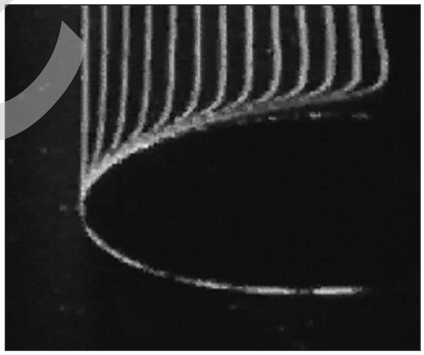

FIGURE 1–14
The development of a velocity profile due to the no-slip condition as a fluid flows over a blunt nose.
"Hunter Rouse: Laminar and Turbulence Flow Film." Copyright IIHR-Hydroscience & Engineering. The University of Iowa. Used by permission.

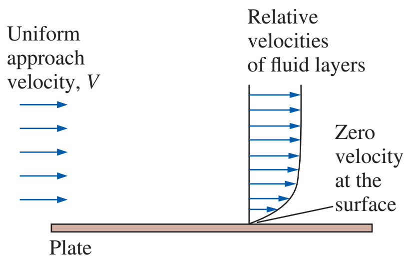

FIGURE 1–15
A fluid flowing over a stationary surface comes to a complete stop at the surface because of the no-slip condition.

1–4 ▪ CLASSIFICATION OF FLUID FLOWS

Earlier we defined *fluid mechanics* as the science that deals with the behavior of fluids at rest or in motion, and the interaction of fluids with solids or other fluids at the boundaries. There is a wide variety of fluid flow problems encountered in practice, and it is usually convenient to classify them on the basis of some common characteristics to make it feasible to study them in groups. There are many ways to classify fluid flow problems, and here we present some general categories.

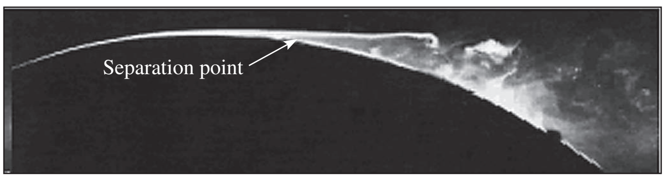

FIGURE 1–16
Flow separation during flow over a curved surface.
From Head, Malcolm R. 1982 in Flow Visualization II, W. Merzkirch. Ed., 399–403, Washington: Hemisphere.

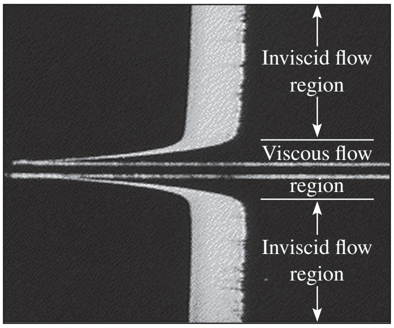

FIGURE 1–17
The flow of an originally uniform fluid stream over a flat plate, and the regions of viscous flow (next to the plate on both sides) and inviscid flow (away from the plate).
Fundamentals of Boundry Layers, National Committee for Fluid Mechanics Films, © Education Development Center.

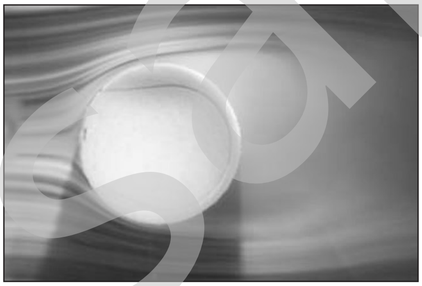

FIGURE 1–18
External flow over a tennis ball, and the turbulent wake region behind.
Courtesy of NASA and Cislunar Aerospace, Inc.

Viscous versus Inviscid Regions of Flow

When two fluid layers move relative to each other, a friction force develops between them and the slower layer tries to slow down the faster layer. This internal resistance to flow is quantified by the fluid property *viscosity,* which is a measure of internal stickiness of the fluid. Viscosity is caused by cohesive forces between the molecules in liquids and by molecular collisions in gases. There is no fluid with zero viscosity, and thus all fluid flows involve viscous effects to some degree. Flows in which the frictional effects are significant are called **viscous flows**. However, in many flows of practical interest, there are *regions* (typically regions not close to solid surfaces) where viscous forces are negligibly small compared to inertial or pressure forces. Neglecting the viscous terms in such **inviscid flow regions** greatly simplifies the analysis without much loss in accuracy.

The development of viscous and inviscid regions of flow as a result of inserting a flat plate parallel into a fluid stream of uniform velocity is shown in Fig. 1–17. The fluid sticks to the plate on both sides because of the no-slip condition, and the thin boundary layer in which the viscous effects are significant near the plate surface is the *viscous flow region*. The region of flow on both sides away from the plate and largely unaffected by the presence of the plate is the *inviscid flow region*.

Internal versus External Flow

A fluid flow is classified as being internal or external, depending on whether the fluid flows in a confined space or over a surface. The flow of an unbounded fluid over a surface such as a plate, a wire, or a pipe is **external flow**. The flow in a pipe or duct is **internal flow** if the fluid is bounded by solid surfaces. Water flow in a pipe, for example, is internal flow, and airflow over a ball or over an exposed pipe during a windy day is external flow (Fig. 1–18). The flow of liquids in a duct is called *open-channel flow* if the duct is only partially filled with the liquid and there is a free surface. The flows of water in rivers and irrigation ditches are examples of such flows.

Internal flows are dominated by the influence of viscosity throughout the flow field. In external flows the viscous effects are limited to boundary layers near solid surfaces and to wake regions downstream of bodies.

Compressible versus Incompressible Flow

A flow is classified as being *compressible* or *incompressible,* depending on the level of variation of density during flow. Incompressibility is an approximation, in which the flow is said to be **incompressible** if the density remains nearly constant throughout. Therefore, the volume of every portion of fluid remains unchanged over the course of its motion when the flow is approximated as incompressible.

The densities of liquids are essentially constant, and thus the flow of liquids is typically incompressible. Therefore, liquids are usually referred to as *incompressible substances.* A pressure of 210 atm, for example, causes the density of liquid water at 1 atm to change by just 1 percent. Gases, on the other hand, are highly compressible. A pressure change of just 0.01 atm, for example, causes a change of 1 percent in the density of atmospheric air.

When analyzing rockets, spacecraft, and other systems that involve high-speed gas flows (Fig. 1–19), the flow speed is often expressed in terms of the dimensionless **Mach number** defined as

$$\mathrm{Ma} = \frac{V}{c} = \frac{\text{Speed of flow}}{\text{Speed of sound}}$$

where c is the **speed of sound** whose value is 346 m/s in air at room temperature at sea level. A flow is called **sonic** when $\mathrm{Ma} = 1$, **subsonic** when $\mathrm{Ma} < 1$, **supersonic** when $\mathrm{Ma} > 1$, and **hypersonic** when $\mathrm{Ma} \gg 1$. Dimensionless parameters are discussed in detail in Chap. 7. Compressible flow is discussed in detail in Chap. 12.

Liquid flows are incompressible to a high level of accuracy, but the level of variation of density in gas flows and the consequent level of approximation made when modeling gas flows as incompressible depends on the Mach number. Gas flows can often be approximated as incompressible if the density changes are under about 5 percent, which is usually the case when $\mathrm{Ma} < 0.3$. Therefore, the compressibility effects of air at room temperature can be neglected at speeds under about 100 m/s. Compressibility effects should never be neglected for supersonic flows, however, since compressible flow phenomena like shock waves occur (Fig. 1–19).

Small density changes of liquids corresponding to large pressure changes can still have important consequences. The irritating "water hammer" in a water pipe, for example, is caused by the vibrations of the pipe generated by the reflection of pressure waves following the sudden closing of the valves.

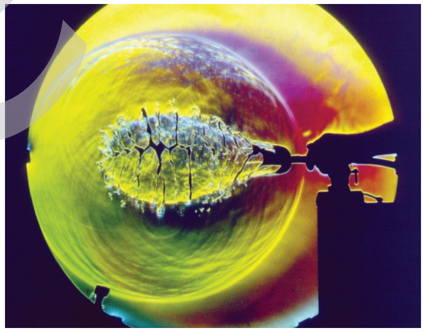

FIGURE 1–19
Schlieren image of the spherical shock wave produced by a bursting ballon at the Penn State Gas Dynamics Lab. Several secondary shocks are seen in the air surrounding the ballon.
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Laminar versus Turbulent Flow

Some flows are smooth and orderly while others are rather chaotic. The highly ordered fluid motion characterized by smooth layers of fluid is called **laminar**. The word *laminar* comes from the movement of adjacent fluid particles together in "laminae." The flow of high-viscosity fluids such as oils at low velocities is typically laminar. The highly disordered fluid motion that typically occurs at high velocities and is characterized by velocity fluctuations is called **turbulent** (Fig. 1–20). The flow of low-viscosity fluids such as air at high velocities is typically turbulent. A flow that alternates between being laminar and turbulent is called **transitional**. The experiments conducted by Osborne Reynolds in the 1880s resulted in the establishment of the dimensionless **Reynolds number**, **Re**, as the key parameter for the determination of the flow regime in pipes (Chap. 8).

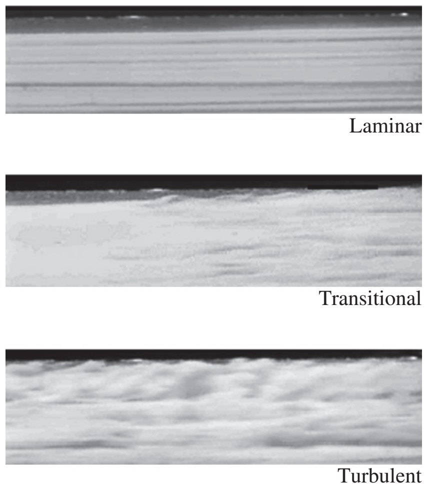

FIGURE 1–20
Laminar, transitional, and turbulent flows over a flat plate.
Courtesy of ONERA. Photo by Werlé.

Natural (or Unforced) versus Forced Flow

A fluid flow is said to be natural or forced, depending on how the fluid motion is initiated. In **forced flow**, a fluid is forced to flow over a surface or in a pipe by external means such as a pump or a fan. In **natural flows**, fluid motion is due to natural means such as the buoyancy effect, which manifests itself as the rise of warmer (and thus lighter) fluid and the fall of cooler (and thus denser) fluid (Fig. 1–21). In solar hot-water systems, for example, the thermosiphoning effect is commonly used to replace pumps by placing the water tank sufficiently above the solar collectors.

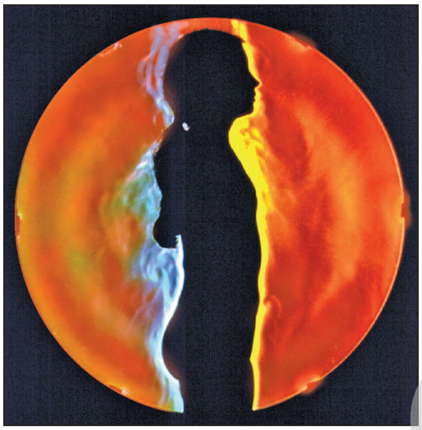

FIGURE 1–21
In this schlieren image of a girl in a swimming suit, the rise of lighter, warmer air adjacent to her body indicates that humans and warm-blooded animals are surrounded by thermal plumes of rising warm air.
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Steady versus Unsteady Flow

The terms *steady* and *uniform* are used frequently in engineering, and thus it is important to have a clear understanding of their meanings. The term **steady** implies *no change of properties, velocity, temperature, etc., at a point with time*. The opposite of steady is **unsteady**. The term **uniform** implies *no change with location* over a specified region. These meanings are consistent with their everyday use (steady girlfriend, uniform distribution, etc.).

The terms *unsteady* and *transient* are often used interchangeably, but these terms are not synonyms. In fluid mechanics, *unsteady* is the most general term that applies to any flow that is not steady, but **transient** is typically used for developing flows. When a rocket engine is fired up, for example, there are transient effects (the pressure builds up inside the rocket engine, the flow accelerates, etc.) until the engine settles down and operates steadily. The term **periodic** refers to the kind of unsteady flow in which the flow oscillates about a steady mean.

Many devices such as turbines, compressors, boilers, condensers, and heat exchangers operate for long periods of time under the same conditions, and they are classified as *steady-flow devices*. (Note that the flow field near the rotating blades of a turbomachine is of course unsteady, but we consider the overall flow field rather than the details at some localities when we classify devices.) During steady flow, the fluid properties can change from point to point within a device, but at any fixed point they remain constant. Therefore, the volume, the mass, and the total energy content of a steady-flow device or flow section remain constant in steady operation. A simple analogy is shown in Fig. 1–22.

Steady-flow conditions can be closely approximated by devices that are intended for continuous operation such as turbines, pumps, boilers, condensers, and heat exchangers of power plants or refrigeration systems. Some cyclic devices, such as reciprocating engines or compressors, do not satisfy the steady-flow conditions since the flow at the inlets and the exits is pulsating and not steady. However, the fluid properties vary with time in a

(*a*) (*b*)

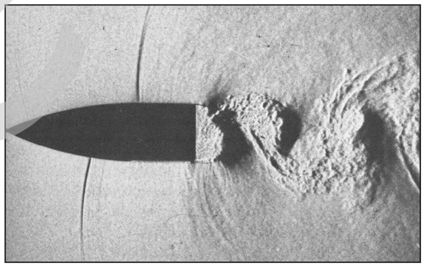
FIGURE 1–22
Comparison of (*a*) instantaneous snapshot of an unsteady flow, and (*b*) long exposure picture of the same flow.
Photos by Eric G. Paterson. Used by permission.

periodic manner, and the flow through these devices can still be analyzed as a steady-flow process by using time-averaged values for the properties.

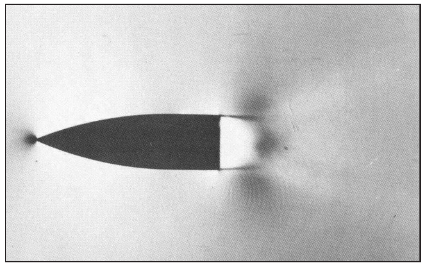
Some fascinating visualizations of fluid flow are provided in the book *An Album of Fluid Motion* by Milton Van Dyke (1982). A nice illustration of an unsteady-flow field is shown in Fig. 1–23, taken from Van Dyke's book. Figure 1–23*a* is an instantaneous snapshot from a high-speed motion picture; it reveals large, alternating, swirling, turbulent eddies that are shed into the periodically oscillating wake from the blunt base of the object. The unsteady wake produces waves that move upstream alternately over the top and bottom surfaces of the airfoil in an unsteady fashion. Figure 1–23*b* shows the *same* flow field, but the film is exposed for a longer time so that the image is time averaged over 12 cycles. The resulting time-averaged flow field appears "steady" since the details of the unsteady oscillations have been lost in the long exposure.

One of the most important jobs of an engineer is to determine whether it is sufficient to study only the time-averaged "steady" flow features of a problem, or whether a more detailed study of the unsteady features is required. If the engineer were interested only in the overall properties of the flow field (such as the time-averaged drag coefficient, the mean velocity, and pressure fields), a time-averaged description like that of Fig. 1–23*b*, time-averaged experimental measurements, or an analytical or numerical calculation of the time-averaged flow field would be sufficient. However, if the engineer were interested in details about the unsteady-flow field, such as flow-induced vibrations, unsteady pressure fluctuations, or the sound waves emitted from the turbulent eddies or the shock waves, a time-averaged description of the flow field would be insufficient.

Most of the analytical and computational examples provided in this textbook deal with steady or time-averaged flows, although we occasionally point out some relevant unsteady-flow features as well when appropriate.



(*a*)



(*b*)

FIGURE 1–23
Oscillating wake of a blunt-based airfoil at Mach number 0.6. Photo (*a*) is an instantaneous image, while photo (*b*) is a long-exposure (time-averaged) image.

(*a*) *Dyment, A., Flodrops, J. P. & Gryson, P. 1982 in* Flow Visualization II, *W. Merzkirch, ed., 331–336. Washington:* Hemisphere. *Used by permission of Arthur Dyment.*

(*b*) *Dyment, A. & Gryson, P. 1978 in* Inst. Mèc. Fluides Lille, *No. 78-5. Used by permission of Arthur Dyment.*

One-, Two-, and Three-Dimensional Flows

A flow field is best characterized by its velocity distribution, and thus a flow is said to be one-, two-, or three-dimensional if the flow velocity varies in one, two, or three primary dimensions, respectively. A typical fluid flow involves a three-dimensional geometry, and the velocity may vary in all three dimensions, rendering the flow three-dimensional [$\vec{V}(x, y, z)$ in rectangular or $\vec{V}(r, \theta, z)$ in cylindrical coordinates]. However, the variation of velocity in certain directions can be small relative to the variation in other directions and can be ignored with negligible error. In such cases, the flow can be modeled conveniently as being one- or two-dimensional, which is easier to analyze.

Consider steady flow of a fluid entering from a large tank into a circular pipe. The fluid velocity everywhere on the pipe surface is zero because of the no-slip condition, and the flow is two-dimensional in the entrance region of the pipe since the velocity changes in both the r- and z-directions, but not in the θ-direction. The velocity profile develops fully and remains unchanged after some distance from the inlet (about 10 pipe diameters in turbulent flow, and typically farther than that in laminar pipe flow, as in Fig. 1–24), and the flow in this region is said to be *fully developed*. The fully developed flow in a circular pipe is *one-dimensional* since the velocity varies in the radial r-direction but not in the angular θ- or axial z-directions, as shown in Fig. 1–24. That is, the velocity profile is the same at any axial z-location, and it is symmetric about the axis of the pipe.

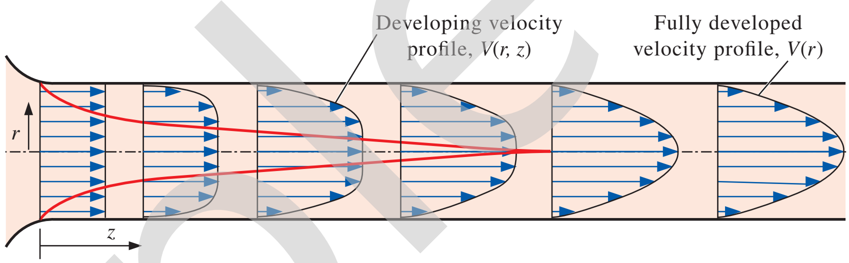


FIGURE 1–24
The development of the velocity profile in a circular pipe. $V = V(r, z)$ and thus the flow is two-dimensional in the entrance region, and becomes one-dimensional downstream when the velocity profile fully develops and remains unchanged in the flow direction, $V = V(r)$.

Note that the dimensionality of the flow also depends on the choice of coordinate system and its orientation. The pipe flow discussed, for example, is one-dimensional in cylindrical coordinates, but two-dimensional in Cartesian coordinates—illustrating the importance of choosing the most appropriate coordinate system. Also note that even in this simple flow, the velocity cannot be uniform across the cross section of the pipe because of the no-slip condition. However, at a well-rounded entrance to the pipe, the velocity profile may be approximated as being nearly uniform across the pipe, since the velocity is nearly constant at all radii except very close to the pipe wall.

A flow may be approximated as *two-dimensional* when the aspect ratio is large and the flow does not change appreciably along the longer dimension. For example, the flow of air over a car antenna can be considered two-dimensional except near its ends since the antenna's length is much greater than its diameter, and the airflow hitting the antenna is fairly uniform (Fig. 1–25).

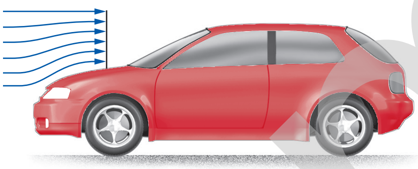


FIGURE 1–25
Flow over a car antenna is approximately two-dimensional except near the top and bottom of the antenna.

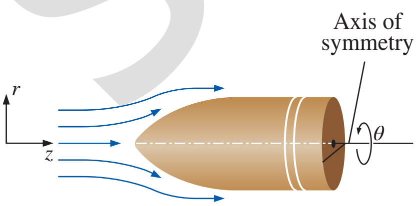


FIGURE 1–26
Axisymmetric flow over a bullet.

EXAMPLE 1–1 Axisymmetric Flow over a Bullet

Consider a bullet piercing through calm air during a short time interval in which the bullet's speed is nearly constant. Determine if the time-averaged airflow over the bullet during its flight is one-, two-, or three-dimensional (Fig. 1–26).

SOLUTION It is to be determined whether airflow over a bullet is one-, two-, or three-dimensional.

Assumptions There are no significant winds and the bullet is not spinning.

Analysis The bullet possesses an axis of symmetry and is therefore an axisymmetric body. The airflow upstream of the bullet is parallel to this axis, and we expect the time-averaged airflow to be rotationally symmetric about the axis—such flows are said to be axisymmetric. The velocity in this case varies with axial distance z and radial distance r, but not with angle θ. Therefore, the time-averaged airflow over the bullet is **two-dimensional**.

Discussion While the time-averaged airflow is axisymmetric, the *instantaneous* airflow is not, as illustrated in Fig. 1–23. In Cartesian coordinates, the flow would be three-dimensional. Finally, many bullets also spin.

Uniform versus Nonuniform Flow

Uniform flow implies that all fluid properties, such as velocity, pressure, temperature, etc., do not vary with position. A wind tunnel test section, for example, is designed such that the air flow is as uniform as possible. Even then, however, the flow does not remain uniform as we approach the wind tunnel walls, due to the no-slip condition and the presence of a boundary layer, as mentioned previously. The flow just downstream of a well-rounded pipe entrance (Fig. 1–24) is nearly uniform, again except for a very thin boundary layer near the wall. In engineering practice, it is common to approximate the flow in ducts and pipes and at inlets and outlets as uniform, even when it is not, for simplicity in calculations. For example, the fully developed pipe flow velocity profile of Fig. 1–24 is certainly not uniform, but for calculation purposes we sometimes approximate it as the uniform profile at the far left of the pipe, which has the same average velocity. Although this makes the calculations easier, it also introduces some errors that require correction factors; these are discussed in Chaps. 5 and 6 for kinetic energy and momentum, respectively.

1–5 ▪ SYSTEM AND CONTROL VOLUME

A **system** is defined as a *quantity of matter or a region in space chosen for study*. The mass or region outside the system is called the **surroundings**. The real or imaginary surface that separates the system from its surroundings is called the **boundary** (Fig. 1–27). The boundary of a system can be *fixed* or *movable*. Note that the boundary is the contact surface shared by both the system and the surroundings. Mathematically speaking, the boundary has zero thickness, and thus it can neither contain any mass nor occupy any volume in space.

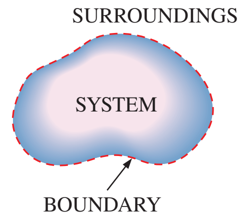


FIGURE 1–27
System, surroundings, and boundary.

Systems may be considered to be *closed* or *open,* depending on whether a fixed mass or a volume in space is chosen for study. A **closed system** (also known as a **control mass** or simply a *system* when the context makes it clear) consists of a fixed amount of mass, and no mass can cross its boundary. But energy, in the form of heat or work, can cross the boundary, and the volume of a closed system does not have to be fixed. If, as a special case, even energy is not allowed to cross the boundary, that system is called an **isolated system**.

Consider the piston–cylinder device shown in Fig. 1–28. Let us say that we would like to find out what happens to the enclosed gas when it is heated. Since we are focusing our attention on the gas, it is our system. The inner surfaces of the piston and the cylinder form the boundary, and since no mass is crossing this boundary, it is a closed system. Notice that energy may cross the boundary, and part of the boundary (the inner surface of the piston, in this case) may move. Everything outside the gas, including the piston and the cylinder, is the surroundings.

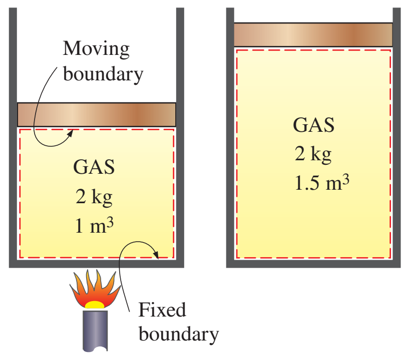
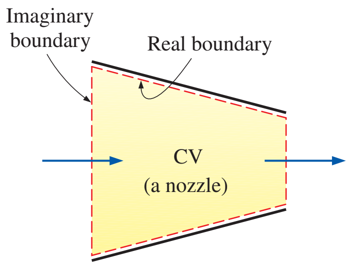


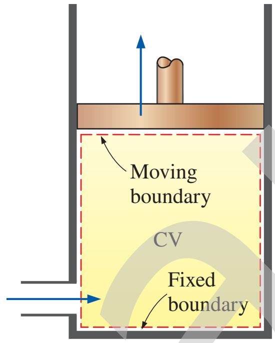
FIGURE 1–28
A closed system with a moving boundary.

An **open system**, or a **control volume**, as it is often called, is a *selected region in space*. It usually encloses a device that involves mass flow such as a compressor, turbine, or nozzle. Flow through these devices is best studied by selecting the region within the device as the control volume. Both mass and energy can cross the boundary (the *control surface*) of a control volume.

A large number of engineering problems involve mass flow in and out of an open system and, therefore, are modeled as *control volumes*. A water heater, a car radiator, a turbine, and a compressor all involve mass flow and should be analyzed as control volumes (open systems) instead of as control masses (closed systems). In general, *any arbitrary region in space* can be selected as a control volume. There are no concrete rules for the



(*a*) A control volume (CV) with real and imaginary boundaries



(*b*) A control volume (CV) with fixed and moving boundaries as well as real and imaginary boundaries

FIGURE 1–29
A control volume may involve fixed, moving, real, and imaginary boundaries.

selection of control volumes, but a wise choice certainly makes the analysis much easier. If we were to analyze the flow of air through a nozzle, for example, a good choice for the control volume would be the region within the nozzle, or perhaps surrounding the entire nozzle.

A control volume can be fixed in size and shape, as in the case of a nozzle, or it may involve a moving boundary, as shown in Fig. 1–29. Most control volumes, however, have fixed boundaries and thus do not involve any moving boundaries. A control volume may also involve heat and work interactions just as a closed system, in addition to mass interaction.

1–6 ▪ IMPORTANCE OF DIMENSIONS AND UNITS

Any physical quantity can be characterized by **dimensions**. The magnitudes assigned to the dimensions are called **units**. Some basic dimensions such as mass m, length L, time t, and temperature T are selected as **primary** or **fundamental dimensions**, while others such as velocity V, energy E, and volume $\mathcal{V}$ are expressed in terms of the primary dimensions and are called **secondary dimensions**, or **derived dimensions**.

A number of unit systems have been developed over the years. Despite strong efforts in the scientific and engineering community to unify the world with a single unit system, two sets of units are still in common use today: the **English system**, which is also known as the *United States Customary System* (USCS), and the metric **SI** (from *Le Système International d' Unités),* which is also known as the *International System.* The SI is a simple and logical system based on a decimal relationship between the various units, and it is being used for scientific and engineering work in most of the industrialized nations, including England. The English system, however, has no apparent systematic numerical base, and various units in this system are related to each other rather arbitrarily (12 in = 1 ft, 1 mile = 5280 ft, 4 qt = 1 gal, etc.), which makes it confusing and difficult to learn. The United States is the only industrialized country that has not yet fully converted to the metric system.

The systematic efforts to develop a universally acceptable system of units dates back to 1790 when the French National Assembly charged the French Academy of Sciences to come up with such a unit system. An early version of the metric system was soon developed in France, but it did not find universal acceptance until 1875 when *The Metric Convention Treaty* was prepared and signed by 17 nations, including the United States. In this international treaty, meter and gram were established as the metric units for length and mass, respectively, and a *General Conference of Weights and Measures* (CGPM) was established that was to meet every six years. In 1960, the CGPM produced the SI, which was based on six fundamental quantities, and their units were adopted in 1954 at the Tenth General Conference of Weights and Measures: *meter* (m) for length, *kilogram* (kg) for mass, *second* (s) for time, *ampere* (A) for electric current, *degree Kelvin* (°K) for temperature, and *candela* (cd) for luminous intensity (amount of light). In 1971, the CGPM added a seventh fundamental quantity and unit: *mole* (mol) for the amount of matter.

Based on the notational scheme introduced in 1967, the degree symbol was officially dropped from the absolute temperature unit, and all unit names were to be written without capitalization even if they were derived from proper names (Table 1–1). However, the abbreviation of a unit was to be capitalized if the unit was derived from a proper name. For example, the SI unit of force, which is named after Sir Isaac Newton (1647–1723), is *newton* (not Newton), and it is abbreviated as N. Also, the full name of a unit may be pluralized, but its abbreviation cannot. For example, the length of an object can be 5 m or 5 meters, *not* 5 ms or 5 meter. Finally, no period is to be used in unit abbreviations unless they appear at the end of a sentence. For example, the proper abbreviation of meter is m (not m.).

The recent move toward the metric system in the United States seems to have started in 1968 when Congress, in response to what was happening in the rest of the world, passed a Metric Study Act. Congress continued to promote a voluntary switch to the metric system by passing the Metric Conversion Act in 1975. A trade bill passed by Congress in 1988 set a September 1992 deadline for all federal agencies to convert to the metric system. However, the deadlines were relaxed later with no clear plans for the future.

As pointed out, the SI is based on a decimal relationship between units. The prefixes used to express the multiples of the various units are listed in Table 1–2. They are standard for all units, and the student is encouraged to memorize some of them because of their widespread use (Fig. 1–30).

TABLE 1–1

The seven fundamental (or primary) dimensions and their units in SI

Dimension	Unit
Length	meter (m)
Mass	kilogram (kg)
Time	second (s)
Temperature	kelvin (K)
Electric current	ampere (A)
Amount of light	candela (cd)
Amount of matter	mole (mol)

TABLE 1–2

Standard prefixes in SI units

Multiple	Prefix
10^{24}	yotta, Y
10^{21}	zetta, Z
10^{18}	exa, E
10^{15}	peta, P
10^{12}	tera, T
10^{9}	giga, G
10^{6}	mega, M
10^{3}	kilo, k
10^{2}	hecto, h
10^{1}	deka, da
10^{-1}	deci, d
10^{-2}	centi, c
10^{-3}	milli, m
10^{-6}	micro, μ
10^{-9}	nano, n
10^{-12}	pico, p
10^{-15}	femto, f
10^{-18}	atto, a
10^{-21}	zepto, z
10^{-24}	yocto, y

Some SI and English Units

In SI, the units of mass, length, and time are the kilogram (kg), meter (m), and second (s), respectively. The respective units in the English system are the pound-mass (lbm), foot (ft), and second (s). The pound symbol *lb* is actually the abbreviation of *libra,* which was the ancient Roman unit of weight. The English retained this symbol even after the end of the Roman occupation of Britain in 410. The mass and length units in the two systems are related to each other by

$$1 \text{ lbm} = 0.45359 \text{ kg}$$

$$1 \text{ ft} = 0.3048 \text{ m}$$

In the English system, force is often considered to be one of the primary dimensions and is assigned a nonderived unit. This is a source of confusion and error that necessitates the use of a dimensional constant (g_c) in many formulas. To avoid this nuisance, we consider force to be a secondary dimension whose unit is derived from Newton's second law, i.e.,

$$\text{Force} = (\text{Mass})\ (\text{Acceleration})$$

or

$$F = ma \tag{1–1}$$

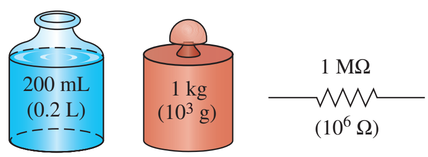


FIGURE 1–30
The SI unit prefixes are used in all branches of engineering.

In SI, the force unit is the newton (N), and it is defined as the *force required to accelerate a mass of 1 kg at a rate of 1 m/s*^{2}. In the English system, the force unit is the **pound-force** (lbf) and is defined as the *force required to*

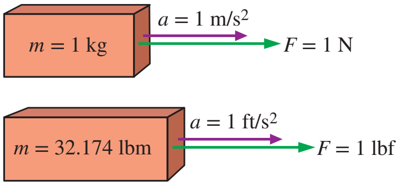


FIGURE 1–31
The definition of the force units.

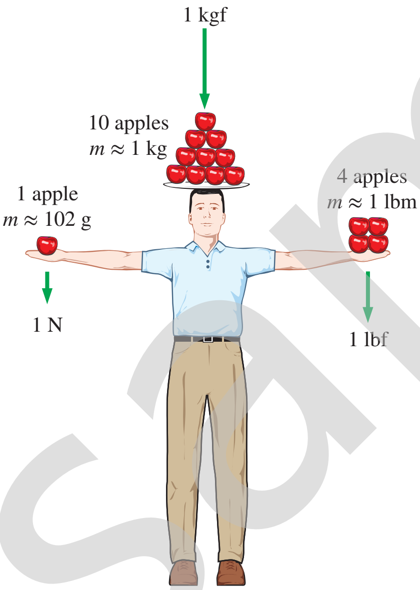


FIGURE 1–32
The relative magnitudes of the force units newton (N), kilogram-force (kgf), and pound-force (lbf).

FIGURE 1–33
A body weighing 150 lbf on earth will weigh only 25 lbf on the moon.

accelerate a mass of 1 slug (32.174 lbm) at a rate of 1 ft/s^2 (Fig. 1–31). That is,

$$1 \text{ N} = 1 \text{ kg·m/s}^2$$

$$1 \text{ lbf} = 32.174 \text{ lbm·ft/s}^2$$

A force of 1 N is roughly equivalent to the weight of a small apple ($m = 102$ g), whereas a force of 1 lbf is roughly equivalent to the weight of four medium apples ($m_{\text{total}} = 454$ g), as shown in Fig. 1–32. Another force unit in common use in many European countries is the *kilogram-force* (kgf), which is the weight of 1 kg mass at sea level (1 kgf = 9.807 N).

The term **weight** is often incorrectly used to express mass, particularly by the "weight watchers." Unlike mass, weight W is a *force*. It is the gravitational force applied to a body, and its magnitude is determined from an equation based on Newton's second law,

$$W = mg \qquad \text{(N)} \tag{1–2}$$

where m is the mass of the body, and g is the local gravitational acceleration (g is 9.807 m/s^2 or 32.174 ft/s^2 at sea level and 45° latitude). An ordinary bathroom scale measures the gravitational force acting on a body. The weight per unit volume of a substance is called the **specific weight** γ and is determined from $\gamma = \rho g$, where ρ is density.

The mass of a body remains the same regardless of its location in the universe. Its weight, however, changes with a change in gravitational acceleration. A body weighs less on top of a mountain since g decreases (by a small amount) with altitude. On the surface of the moon, an astronaut weighs about one-sixth of what she or he normally weighs on earth (Fig. 1–33).

At sea level a mass of 1 kg weighs 9.807 N, as illustrated in Fig. 1–34. A mass of 1 lbm, however, weighs 1 lbf, which misleads people to believe that pound-mass and pound-force can be used interchangeably as pound (lb), which is a major source of error in the English system.

It should be noted that the *gravity force* acting on a mass is due to the *attraction* between the masses, and thus it is proportional to the magnitudes of the masses and inversely proportional to the square of the distance between them. Therefore, the gravitational acceleration g at a location depends on *latitude,* the *distance* to the earth, and to a lesser extent, the positions of the moon and the sun. The value of g varies with location from 9.8295 m/s^2 at 4500 m below sea level to 7.3218 m/s^2 at 100,000 m above sea level. However, at altitudes up to 30,000 m, the variation of g from the sea-level value of 9.807 m/s^2 is less than 1 percent. Therefore, for most practical purposes, the gravitational acceleration can be assumed to be *constant* at 9.807 m/s^2, often rounded to 9.81 m/s^2. It is interesting to note that the value of g increases with distance below sea level, reaches a maximum at about 4500 m below sea level, and then starts decreasing. (What do you think the value of g is at the center of the earth?)

The primary cause of confusion between mass and weight is that mass is usually measured *indirectly* by measuring the *gravity force* it exerts. This approach also assumes that the forces exerted by other effects such as air buoyancy and fluid motion are negligible. This is like measuring the distance to a star by measuring its red shift, or measuring the altitude of an airplane by measuring barometric pressure. Both of these are also indirect measurements. The correct *direct* way of measuring mass is to compare it to a known mass. This is cumbersome, however, and it is mostly used for calibration and measuring precious metals.

Work, which is a form of energy, can simply be defined as force times distance; therefore, it has the unit "newton-meter (N·m)," which is called a **joule** (J). That is,

$$1 \text{ J} = 1 \text{ N·m} \tag{1–3}$$

A more common unit for energy in SI is the kilojoule (1 kJ = 10^3 J). In the English system, the energy unit is the **Btu** (British thermal unit), which is defined as the energy required to raise the temperature of 1 lbm of water at 68°F by 1°F. In the metric system, the amount of energy needed to raise the temperature of 1 g of water at 14.5°C by 1°C is defined as 1 **calorie** (cal), and 1 cal = 4.1868 J. The magnitudes of the kilojoule and Btu are very nearly the same (1 Btu = 1.0551 kJ). Here is a good way to get a feel for these units: If you light a typical match and let it burn itself out, it yields approximately one Btu (or one kJ) of energy (Fig. 1–35).

The unit for time rate of energy is joule per second (J/s), which is called a **watt** (W). In the case of work, the time rate of energy is called *power*. A commonly used unit of power is horsepower (hp), which is equivalent to 745.7 W. Electrical energy typically is expressed in the unit kilowatt-hour (kWh), which is equivalent to 3600 kJ. An electric appliance with a rated power of 1 kW consumes 1 kWh of electricity when running continuously for one hour. When dealing with electric power generation, the units kW and kWh are often confused. Note that kW or kJ/s is a unit of power, whereas kWh is a unit of energy. Therefore, statements like "the new wind turbine will generate 50 kW of electricity per year" are meaningless and incorrect. A correct statement should be something like "the new wind turbine with a rated power of 50 kW will generate 120,000 kWh of electricity per year."

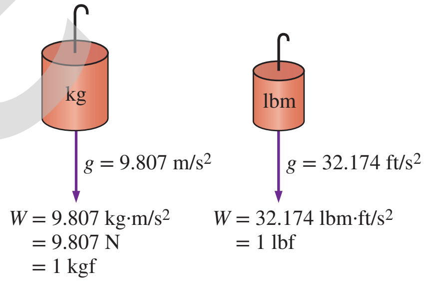


FIGURE 1–34
The weight of a unit mass at sea level.

FIGURE 1–35
A typical match yields about one Btu (or one kJ) of energy if completely burned.
Photo by John M. Cimbala.

Dimensional Homogeneity

We all know that you cannot add apples and oranges. But we somehow manage to do it (by mistake, of course). In engineering, all equations must be *dimensionally homogeneous.* That is, every term in an equation must have the same dimensions. If, at some stage of an analysis, we find ourselves in a position to add two quantities that have different dimensions or units, it is a clear indication that we have made an error at an earlier stage. So checking dimensions (or units) can serve as a valuable tool to spot errors.

FIGURE 1–36
A wind turbine, as discussed in Example 1–2.
Photo by Andrew Cimbala.

EXAMPLE 1–2 **Electric Power Generation by a Wind Turbine**

A school is paying \$0.09/kWh for electric power. To reduce its power bill, the school installs a wind turbine (Fig. 1–36) with a rated power of 30 kW. If the turbine operates 2200 hours per year at the rated power, determine the amount of electric power generated by the wind turbine and the money saved by the school per year.

SOLUTION A wind turbine is installed to generate electricity. The amount of electric energy generated and the money saved per year are to be determined.

Analysis The wind turbine generates electric energy at a rate of 30 kW or 30 kJ/s. Then the total amount of electric energy generated per year becomes

$$\begin{aligned}\text{Total energy} &= (\text{Energy per unit time})(\text{Time interval})\\ &= (30\text{ kW})(2200\text{ h})\\ &= \mathbf{66{,}000\text{ kWh}}\end{aligned}$$

The money saved per year is the monetary value of this energy determined as

$$\begin{aligned}\text{Money saved} &= (\text{Total energy})(\text{Unit cost of energy})\\ &= (66{,}000\text{ kWh})(\$0.09/\text{kWh})\\ &= \mathbf{\$5940}\end{aligned}$$

Discussion The annual electric energy production also could be determined in kJ by unit manipulations as

$$\text{Total energy} = (30\text{ kW})(2200\text{ h})\left(\frac{3600\text{ s}}{1\text{ h}}\right)\left(\frac{1\text{ kJ/s}}{1\text{ kW}}\right) = 2.38\times10^{8}\text{ kJ}$$

which is equivalent to 66,000 kWh (1 kWh = 3600 kJ).

We all know from experience that units can give terrible headaches if they are not used carefully in solving a problem. However, with some attention and skill, units can be used to our advantage. They can be used to check formulas; sometimes they can even be used to *derive* formulas, as explained in the following example.

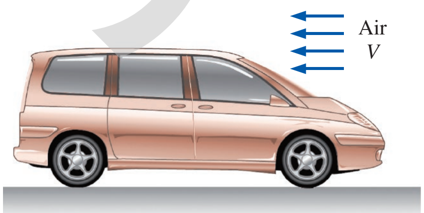


FIGURE 1–37
Schematic for Example 1–3.

EXAMPLE 1–3 Obtaining Formulas from Unit Consideration

The drag force exerted on a car by air depends on a dimensionless drag coefficient, the density of air, the car velocity, and the frontal area of the car (Fig. 1–37). That is, $F_D = F_D\,(C_{\text{drag}}, A_{\text{front}}, \rho, V)$. Based on unit considerations alone, obtain a relation for the drag force.

SOLUTION A relation for the air drag exerted on a car is to be obtained in terms of the drag coefficient, the air density, the car velocity, and the frontal area of the car.

Analysis The drag force depends on a dimensionless drag coefficient, the air density, the car velocity, and the frontal area. Also, the unit of force F is newton N, which is equivalent to kg·m/s^2. Therefore, the independent quantities should be arranged such that we end up with the unit kg·m/s^2 for the drag force. Putting the given information into perspective, we have

$$F_D[\text{kg}\cdot\text{m/s}^2] = C_{\text{drag}}[-],\ A_{\text{front}}[\text{m}^2],\ \rho[\text{kg/m}^3],\ \text{and } V[\text{m/s}]$$

It is obvious that the only way to end up with the unit "kg·m/s^2" for drag force is to multiply density with the square of the velocity and the frontal area, with the drag coefficient serving as the constant of proportionality. Therefore, the desired relation is

$$F_D = C_{\text{drag}}\rho A_{\text{front}}V^2 \ \leftrightarrow\ \text{kg}\cdot\text{m/s}^2 = [\text{kg/m}^3]\,[\text{m}^2]\,[\text{m}^2/\text{s}^2]$$

Discussion Note that the drag coefficient is dimensionless, so we cannot be sure whether it goes in the numerator or denominator, or has some exponent, etc. Common sense dictates, however, that the drag force should be linearly proportional to the drag coefficient.

You should keep in mind that a formula that is not dimensionally homogeneous is definitely wrong (Fig. 1–38), but a dimensionally homogeneous formula is not necessarily right.

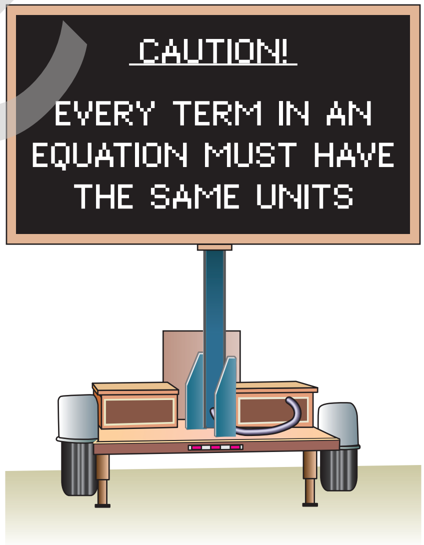


FIGURE 1–38
Always check the units in your calculations.

Unity Conversion Ratios

Just as all nonprimary dimensions can be formed by suitable combinations of primary dimensions, *all nonprimary units (**secondary units**) can be formed by combinations of primary units.* Force units, for example, can be expressed as

$$\text{N} = \text{kg}\,\frac{\text{m}}{\text{s}^2} \quad \text{and} \quad \text{lbf} = 32.174\ \text{lbm}\,\frac{\text{ft}}{\text{s}^2}$$

They can also be expressed more conveniently as **unity conversion ratios** as

$$\frac{\text{N}}{\text{kg·m/s}^2} = 1 \quad \text{and} \quad \frac{\text{lbf}}{32.174\ \text{lbm·ft/s}^2} = 1$$

Unity conversion ratios are identically equal to 1 and are unitless, and thus such ratios (or their inverses) can be inserted conveniently into any calculation to properly convert units (Fig. 1–39). You are encouraged to always use unity conversion ratios such as those given here when converting units. Some textbooks insert the archaic gravitational constant g_c defined as $g_c = 32.174$ lbm·ft/lbf·s^2 = kg·m/N·s^2 = 1 into equations in order to force units to match. This practice leads to unnecessary confusion and is strongly discouraged by the present authors. We recommend that you instead use unity conversion ratios.

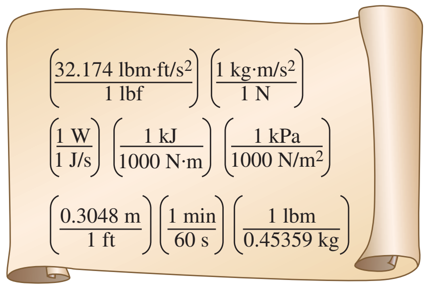


FIGURE 1–39
Every unity conversion ratio (as well as its inverse) is exactly equal to one. Shown here are a few commonly used unity conversion ratios, each within its own set of parentheses.

EXAMPLE 1–4 The Weight of One Pound-Mass

Using unity conversion ratios, show that 1.00 lbm weighs 1.00 lbf on earth (Fig. 1–40).

SOLUTION A mass of 1.00 lbm is subjected to standard earth gravity. Its weight in lbf is to be determined.
Assumptions Standard sea-level conditions are assumed.
Properties The gravitational constant is $g = 32.174$ ft/s^2.
Analysis We apply Newton's second law to calculate the weight (force) that corresponds to the known mass and acceleration. The weight of any object is equal to its mass times the local value of gravitational acceleration. Thus,

$$W = mg = (1.00\ \text{lbm})(32.174\ \text{ft/s}^2)\left(\frac{1\ \text{lbf}}{32.174\ \text{lbm·ft/s}^2}\right) = \mathbf{1.00\ lbf}$$

Discussion The quantity in large parentheses in this equation is a unity conversion ratio. Mass is the same regardless of its location. However, on some other planet with a different value of gravitational acceleration, the weight of 1 lbm would differ from that calculated here.

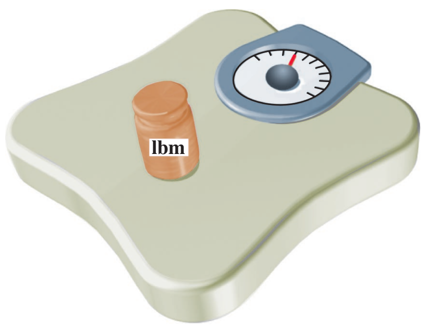


FIGURE 1–40
A mass of 1 lbm weighs 1 lbf on earth.

FIGURE 1–41
A quirk in the metric system of units.

When you buy a box of breakfast cereal, the printing may say "Net weight: One pound (454 grams)." (See Fig. 1–41.) Technically, this means that the cereal inside the box weighs 1.00 lbf on earth and has a *mass* of 453.6 g (0.4536 kg). Using Newton's second law, the actual weight of the cereal on earth is

$$W = mg = (453.6 \text{ g})(9.81 \text{ m/s}^2)\left(\frac{1 \text{ N}}{1 \text{ kg·m/s}^2}\right)\left(\frac{1 \text{ kg}}{1000 \text{ g}}\right) = 4.49 \text{ N}$$

1–7 ▪ MODELING IN ENGINEERING

An engineering device or process can be studied either *experimentally* (testing and taking measurements) or *analytically* (by analysis or calculations). The experimental approach has the advantage that we deal with the actual physical system, and the desired quantity is determined by measurement, within the limits of experimental error. However, this approach is expensive, time-consuming, and often impractical. Besides, the system we are studying may not even exist. For example, the entire heating and plumbing systems of a building must usually be sized *before* the building is actually built on the basis of the specifications given. The analytical approach (including the numerical approach) has the advantage that it is fast and inexpensive, but the results obtained are subject to the accuracy of the assumptions, approximations, and idealizations made in the analysis. In engineering studies, often a good compromise is reached by reducing the choices to just a few by analysis, and then verifying the findings experimentally.

The descriptions of most scientific problems involve equations that relate the changes in some key variables to each other. Usually the smaller the increment chosen in the changing variables, the more general and accurate the description. In the limiting case of infinitesimal or differential changes in variables, we obtain *differential equations* that provide precise mathematical formulations for the physical principles and laws by representing the rates of change as *derivatives*. Therefore, differential equations are used to investigate a wide variety of problems in sciences and engineering (Fig. 1–42). However, many problems encountered in practice can be solved without resorting to differential equations and the complications associated with them.

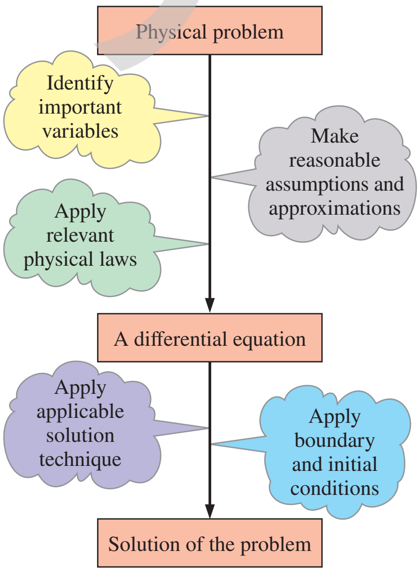


FIGURE 1–42
Mathematical modeling of physical problems.

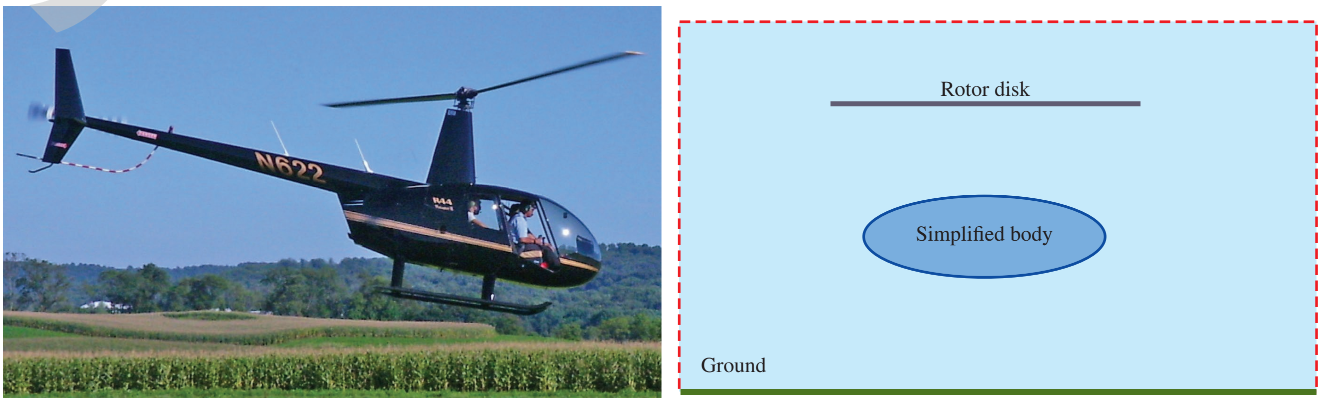
The study of physical phenomena involves two important steps. In the first step, all the variables that affect the phenomena are identified, reasonable assumptions and approximations are made, and the interdependence of these variables is studied. The relevant physical laws and principles are invoked, and the problem is formulated mathematically. The equation itself is very instructive as it shows the degree of dependence of some variables on others, and the relative importance of various terms. In the second step, the problem is solved using an appropriate approach, and the results are interpreted.

Many processes that seem to occur in nature randomly and without any order are, in fact, being governed by some visible or not-so-visible physical laws. Whether we notice them or not, these laws are there, governing consistently and predictably over what seem to be ordinary events. Most of these laws are well defined and well understood by scientists. This makes it possible to predict the course of an event before it actually occurs or to study various aspects of an event mathematically without actually running expensive and time-consuming experiments. This is where the power of analysis lies. Very accurate results to meaningful practical problems can be obtained with relatively little effort by using a suitable and realistic mathematical model. The preparation of such models requires an adequate knowledge of the natural phenomena involved and the relevant laws, as well as sound judgment. An unrealistic model will obviously give inaccurate and thus unacceptable results.

An analyst working on an engineering problem often finds himself or herself in a position to make a choice between a very accurate but complex model, and a simple but not-so-accurate model. The right choice depends on the situation at hand. The right choice is usually the simplest model that yields satisfactory results (Fig. 1–43). Also, it is important to consider the actual operating conditions when selecting equipment.

Preparing very accurate but complex models is usually not so difficult. But such models are not much use to an analyst if they are very difficult and time-consuming to solve. At the minimum, the model should reflect the essential features of the physical problem it represents. There are many significant real-world problems that can be analyzed with a simple model. But it should always be kept in mind that the results obtained from an analysis are at best as accurate as the assumptions made in simplifying the problem. Therefore, the solution obtained should not be applied to situations for which the original assumptions do not hold.

A solution that is not quite consistent with the observed nature of the problem indicates that the mathematical model used is too crude. In that



(*a*) Actual engineering problem

(*b*) Minimum essential model of the engineering problem

FIGURE 1–43
Simplified models are often used in fluid mechanics to obtain approximate solutions to difficult engineering problems. Here, the helicopter's rotor is modeled by a disk, across which is imposed a sudden change in pressure. The helicopter's body is modeled by a simple ellipsoid. This simplified model yields the essential features of the overall air flow field in the vicinity of the ground.

(a) Photo by John M. Cimbala.

case, a more realistic model should be prepared by eliminating one or more of the questionable assumptions. This will result in a more complex problem that, of course, is more difficult to solve. Thus any solution to a problem should be interpreted within the context of its formulation.

1–8 ▪ PROBLEM-SOLVING TECHNIQUE

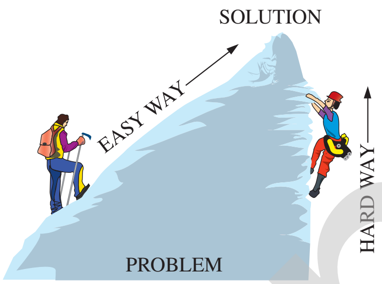


FIGURE 1–44
A step-by-step approach can greatly simplify problem solving.

The first step in learning any science is to grasp the fundamentals and to gain a sound knowledge of it. The next step is to master the fundamentals by testing this knowledge. This is done by solving significant real-world problems. Solving such problems, especially complicated ones, requires a systematic approach. By using a step-by-step approach, an engineer can reduce the solution of a complicated problem into the solution of a series of simple problems (Fig. 1–44). When you are solving a problem, we recommend that you use the following steps zealously as applicable. This will help you avoid some of the common pitfalls associated with problem solving.

Step 1: Problem Statement

In your own words, briefly state the problem, the key information given, and the quantities to be found. This is to make sure that you understand the problem and the objectives before you attempt to solve the problem.

Step 2: Schematic

Draw a realistic sketch of the physical system involved, and list the relevant information on the figure. The sketch does not have to be something elaborate, but it should resemble the actual system and show the key features. Indicate any energy and mass interactions with the surroundings. Listing the given information on the sketch helps one to see the entire problem at once. Also, check for properties that remain constant during a process (such as temperature during an isothermal process), and indicate them on the sketch.

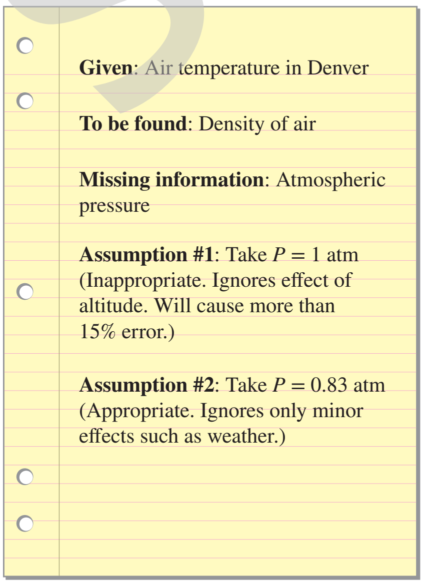


FIGURE 1–45
The assumptions made while solving an engineering problem must be reasonable and justifiable.

Step 3: Assumptions and Approximations

State any appropriate assumptions and approximations made to simplify the problem to make it possible to obtain a solution. Justify the questionable assumptions. Assume reasonable values for missing quantities that are necessary. For example, in the absence of specific data for atmospheric pressure, it can be taken to be 1 atm. However, it should be noted in the analysis that the atmospheric pressure decreases with increasing elevation. For example, it drops to 0.83 atm in Denver (elevation 1610 m) (Fig. 1–45).

Step 4: Physical Laws

Apply all the relevant basic physical laws and principles (such as the conservation of mass), and reduce them to their simplest form by utilizing the assumptions made. However, the region to which a physical law is applied must be clearly identified first. For example, the increase in speed of water flowing through a nozzle is analyzed by applying conservation of mass between the inlet and outlet of the nozzle.

Step 5: Properties

Determine the unknown properties at known states necessary to solve the problem from property relations or tables. List the properties separately, and indicate their source, if applicable.

Step 6: Calculations

Substitute the known quantities into the simplified relations and perform the calculations to determine the unknowns. Pay particular attention to the units and unit cancellations, and remember that a dimensional quantity without a unit is meaningless. Also, don't give a false implication of high precision by copying all the digits from the screen of the calculator—round the final results to an appropriate number of significant digits (Section 1–10).

Step 7: Reasoning, Verification, and Discussion

Check to make sure that the results obtained are reasonable and intuitive, and verify the validity of the questionable assumptions. Repeat the calculations that resulted in unreasonable values. For example, under the same test conditions the aerodynamic drag acting on a car should *not* increase after streamlining the shape of the car (Fig. 1–46).

Also, point out the significance of the results, and discuss their implications. State the conclusions that can be drawn from the results, and any recommendations that can be made from them. Emphasize the limitations under which the results are applicable, and caution against any possible misunderstandings and using the results in situations where the underlying assumptions do not apply. For example, if you determined that using a larger-diameter pipe in a proposed pipeline will cost an additional $5000 in materials, but it will reduce the annual pumping costs by $3000, indicate that the larger-diameter pipeline will pay for its cost differential from the electricity it saves in less than two years. However, also state that only additional material costs associated with the larger-diameter pipeline are considered in the analysis.

Keep in mind that the solutions you present to your instructors, and any engineering analysis presented to others, is a form of communication. Therefore neatness, organization, completeness, and visual appearance are of utmost importance for maximum effectiveness (Fig. 1–47). Besides, neatness also serves as a great checking tool since it is very easy to spot errors and inconsistencies in neat work. Carelessness and skipping steps to save time often end up costing more time and unnecessary anxiety.

The approach described here is used in the solved example problems without explicitly stating each step, as well as in the Solutions Manual of this text. For some problems, some of the steps may not be applicable or necessary. For example, often it is not practical to list the properties separately. However, we cannot overemphasize the importance of a logical and orderly approach to problem solving. Most difficulties encountered while solving a problem are not due to a lack of knowledge; rather, they are due to a lack of organization. You are strongly encouraged to follow these steps in problem solving until you develop your own approach that works best for you.

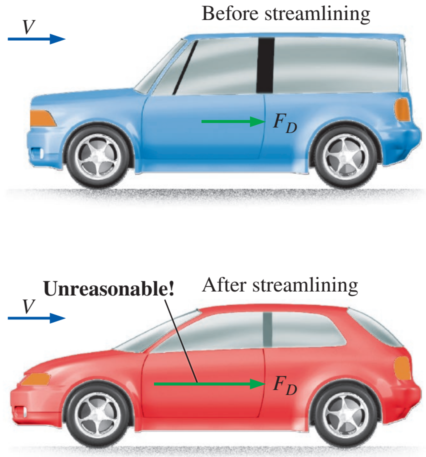


FIGURE 1–46
The results obtained from an engineering analysis must be checked for reasonableness.

FIGURE 1–47
Neatness and organization are highly valued by employers.

1–9 ▪ ENGINEERING SOFTWARE PACKAGES

You may be wondering why we are about to undertake an in-depth study of the fundamentals of another engineering science. After all, almost all such problems we are likely to encounter in practice can be solved using one of several sophisticated software packages readily available in the market today. These software packages not only give the desired numerical results, but also supply the outputs in colorful graphical form for impressive presentations. It is unthinkable to practice engineering today without using some of these packages. This tremendous computing power available to us at the touch of a button is both a blessing and a curse. It certainly enables engineers to solve problems easily and quickly, but it also opens the door for abuses and misinformation. In the hands of poorly educated people, these software packages are as dangerous as sophisticated powerful weapons in the hands of poorly trained soldiers.

Thinking that a person who can use the engineering software packages without proper training in the fundamentals can practice engineering is like thinking that a person who can use a wrench can work as a car mechanic. If it were true that the engineering students do not need all these fundamental courses they are taking because practically everything can be done by computers quickly and easily, then it would also be true that the employers would no longer need high-salaried engineers since any person who knows how to use a word-processing program can also learn how to use those software packages. However, the statistics show that the need for engineers is on the rise, not on the decline, despite the availability of these powerful packages.

FIGURE 1–48
An excellent word-processing program does not make a person a good writer; it simply makes a good writer a more efficient writer.
© Caia Images/Glow Images RF

We should always remember that all the computing power and the engineering software packages available today are just *tools,* and tools have meaning only in the hands of masters. Having the best word-processing program does not make a person a good writer, but it certainly makes the job of a good writer much easier and makes the writer more productive (Fig. 1–48). Hand calculators did not eliminate the need to teach our children how to add or subtract, and sophisticated medical software packages did not take the place of medical school training. Neither will engineering software packages replace the traditional engineering education. They will simply cause a shift in emphasis in the courses from mathematics to physics. That is, more time will be spent in the classroom discussing the physical aspects of the problems in greater detail, and less time on the mechanics of solution procedures.

All these marvelous and powerful tools available today put an extra burden on today's engineers. They must still have a thorough understanding of the fundamentals, develop a "feel" of the physical phenomena, be able to put the data into proper perspective, and make sound engineering judgments, just like their predecessors. However, they must do it much better, and much faster, using more realistic models because of the powerful tools available today. The engineers in the past had to rely on hand calculations, slide rules, and later hand calculators and computers. Today they rely on software packages. The easy access to such power and the possibility of a simple misunderstanding or misinterpretation causing great damage make it more important today than ever to have solid training in the fundamentals of engineering. In this text we make an extra effort to put the emphasis on developing an intuitive and physical understanding of natural phenomena instead of on the mathematical details of solution procedures.

Equation Solvers

You are probably familiar with the equation solving capabilities of spreadsheets such as Microsoft Excel. Despite its simplicity, Excel is commonly used in solving systems of equations in engineering as well as finance. It enables the user to conduct parametric studies, plot the results, and ask "what if" questions. It can also solve simultaneous equations if properly set up. There are also numerous sophisticated equation solvers commonly used in engineering practice such as the Engineering Equation Solver (EES) which is a program that easily solves systems of linear or nonlinear algebraic or differential equations numerically. It has a large library of built-in thermodynamic property functions as well as mathematical functions, and allows the user to supply additional property data.

Unlike some software packages, equation solvers do not solve engineering problems; they only solve the equations supplied by the user. Therefore, the user must understand the problem and formulate it by applying any relevant physical laws and relations. Equation solvers save the user considerable time and effort by simply solving the resulting mathematical equations. This makes it possible to attempt significant engineering problems not suitable for hand calculations and to conduct parametric studies quickly and conveniently.

EXAMPLE 1–5 Solving a System of Equations Numerically

The difference of two numbers is 4, and the sum of the squares of these two numbers is equal to the sum of the numbers plus 20. Determine these two numbers.

SOLUTION Relations are given for the difference and the sum of the squares of two numbers. The two numbers are to be determined.

Analysis We first solve the problem using EES. We start the EES program by double-clicking on its icon, open a new file, and type the following on the blank screen that appears:

$$x - y = 4$$
$$x\,\hat{}\,2 + y\,\hat{}\,2 = x + y + 20$$

which is an exact mathematical expression of the problem statement with x and y denoting the unknown numbers. The solution to this system of equations (one linear and one nonlinear) with two unknowns is obtained by a single click on the "calculator" icon on the taskbar. It gives (Fig. 1–49)

$$\mathbf{x = 5} \quad \text{and} \quad \mathbf{y = 1}$$

We now solve the same problem using Excel. Start Excel. File/Options/Add-Ins/Solver Add-In/OK, where the underline means to click on that option and the slash separates each sequential option. Choose a cell for x and a cell for y and enter initial guesses there (we chose cells C25 and D25 and guessed 0.5 and 0.5). We must rewrite the two equations so that no variables are on

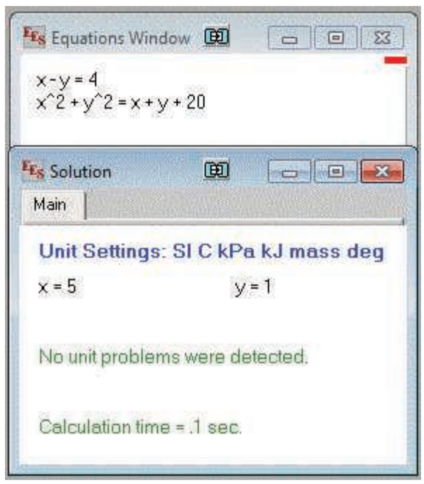


FIGURE 1–49
EES screen images for Example 1–5.

	A	B	C	D
19		Equation		RHS of equation
20		x-y=4		=C25-D25
21		x^2+y^2-x-y=20		=C25^2 + D25^2 -C25-D25
23		Variable values:		
24			x	y
25			0.5	0.5

(*a*)

	A	B	C	D
19		Equation		RHS of equation
20		x-y=4		4
21		x^2+y^2-x-y=20		20
23		Variable values:		
24			x	y
25			5	1

(*b*)

FIGURE 1–50
Excel screen images for Example 1–5. (*a*) Equations, with initial guesses highlighted. (*b*) Final results after using Excel's Solver, with converged values highlighted.

the right-hand side (RHS): $x - y = 4$ and $x^2 + y^2 - x - y = 20$. Choose a cell for the RHS of each equation and enter the formula there (we chose cells D20 and D21; see the equations in Fig. 1–50*a*). Data/Solver. Set the cell for the RHS of the first equation (D20) as the "Objective" with a value of 4, set the cells for x and y (C25:D25) as those subject to constraints, and set the constraint such that the cell for the RHS of the second equation (D21) must equal 20. Solve/OK. The solution iterates to the correct final values of $x = 5$ and $y = 1$, respectively (Fig. 1–50*b*). *Note:* For better convergence, the precision, number of allowed iterations, etc. can be changed in Data/Solver/Options.

Discussion Note that all we did is formulate the problem as we would on paper; EES or Excel took care of all the mathematical details of the solution. Also note that equations can be linear or nonlinear, and they can be entered in any order with unknowns on either side. Friendly equation solvers such as EES allow the user to concentrate on the physics of the problem without worrying about the mathematical complexities associated with the solution of the resulting system of equations.

CFD Software

Computational fluid dynamics (CFD) is used extensively in engineering and research, and we discuss CFD in detail in Chap. 15. We also show example solutions from CFD throughout the textbook since CFD graphics are great for illustrating flow streamlines, velocity, and pressure distributions, etc. beyond what we are able to visualize in the laboratory (Fig. 1–51). However, because there are several different commercial CFD packages available for users, and student access to these codes is highly dependent on departmental licenses, we do not provide end-of-chapter CFD problems that are tied to any particular CFD package. Instead, we provide some general CFD problems in Chap. 15, and we also maintain a website (see link at www.mhhe.com/cengel) containing CFD problems that can be solved with a number of different CFD programs. Students are encouraged to work through some of these problems to become familiar with CFD.

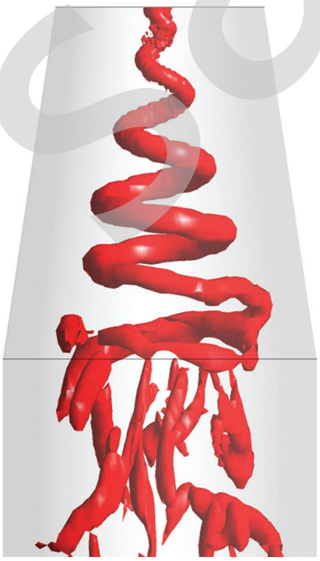


FIGURE 1–51
The unsteady vortex rope formed in the draft tube of a model Francis turbine operating at a discharge coefficient of 0.34. Rope simulated using the commercial CFD software, ANSYS-FLUENT. Shown are isocontours of swirling strength.
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1–10 ▪ ACCURACY, PRECISION, AND SIGNIFICANT DIGITS

In engineering calculations, the supplied information is not known to more than a certain number of significant digits, usually three digits. Consequently, the results obtained cannot possibly be precise to more significant digits. Reporting results in more significant digits falsely implies greater precision than exists, and it should be avoided.

Regardless of the system of units employed, engineers must be aware of three principles that govern the proper use of numbers: accuracy, precision, and significant digits. For engineering measurements, they are defined as follows:

- **Accuracy error** (*inaccuracy*) is the value of one reading minus the true value. In general, accuracy of a set of measurements refers to the closeness of the average reading to the true value. Accuracy is generally associated with repeatable, fixed errors.

- **Precision error** is the value of one reading minus the average of readings. In general, precision of a set of measurements refers to the fineness of the resolution and the repeatability of the instrument. Precision is generally associated with unrepeatable, random errors.
- **Significant digits** are digits that are relevant and meaningful.

A measurement or calculation can be very precise without being very accurate, and vice versa. For example, suppose the true value of wind speed is 25.00 m/s. Two anemometers A and B take five wind speed readings each:

Anemometer A: 25.50, 25.69, 25.52, 25.58, and 25.61 m/s. Average of all readings = 25.58 m/s.

Anemometer B: 26.3, 24.5, 23.9, 26.8, and 23.6 m/s. Average of all readings = 25.02 m/s.

Clearly, anemometer A is more precise, since none of the readings differs by more than 0.11 m/s from the average. However, the average is 25.58 m/s, 0.58 m/s greater than the true wind speed; this indicates significant **bias error**, also called **constant error** or **systematic error**. On the other hand, anemometer B is not very precise, since its readings swing wildly from the average; but its overall average is much closer to the true value. Hence, anemometer B is more accurate than anemometer A, at least for this set of readings, even though it is less precise. The difference between accuracy and precision can be illustrated effectively by analogy to shooting arrows at a target, as sketched in Fig. 1–52. Shooter A is very precise, but not very accurate, while shooter B has better overall accuracy, but less precision.

Many engineers do not pay proper attention to the number of significant digits in their calculations. The least significant numeral in a number implies the precision of the measurement or calculation. For example, a result written as 1.23 (three significant digits) *implies* that the result is precise to within one digit in the second decimal place; i.e., the number is somewhere between 1.22 and 1.24. Expressing this number with any more digits would be misleading. The number of significant digits is most easily evaluated when the number is written in exponential notation; the number of significant digits can then simply be counted, including zeroes. Alternatively, the least significant digit can be underlined to indicate the author's intent. Some examples are shown in Table 1–3.

When performing calculations or manipulations of several parameters, the final result is generally only as precise as the least precise parameter in the problem. For example, suppose A and B are multiplied to obtain C. If $A = 2.3601$ (five significant digits), and $B = 0.34$ (two significant digits), then $C = 0.80$ (only two digits are significant in the final result). Note that most students are tempted to write $C = 0.802434$, with six significant digits, since that is what is displayed on a calculator after multiplying these two numbers.

Let's analyze this simple example carefully. Suppose the exact value of B is 0.33501, which is read by the instrument as 0.34. Also suppose A is exactly 2.3601, as measured by a more accurate and precise instrument. In this case, $C = A \times B = 0.79066$ to five significant digits. Note that our first answer, $C = 0.80$ is off by one digit in the second decimal place. Likewise, if B is 0.34499, and is read by the instrument as 0.34, the product of A and B would be 0.81421 to five significant digits. Our original answer of 0.80

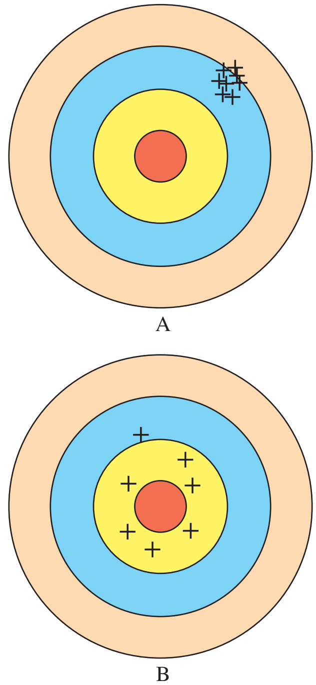


FIGURE 1–52
Illustration of accuracy versus precision. Shooter A is more precise, but less accurate, while shooter B is more accurate, but less precise.

TABLE 1–3
Significant digits

Number	Exponential Notation	Number of Significant Digits
12.3	1.23×10^1	3
$12\underline{3},000$	1.23×10^5	3
0.00123	1.23×10^{-3}	3
$40,\underline{3}00$	4.03×10^4	3
40,300	4.0300×10^4	5
0.005600	5.600×10^{-3}	4
0.0056	5.6×10^{-3}	2
0.006	$6. \times 10^{-3}$	1

is again off by one digit in the second decimal place. The main point here is that 0.80 (to two significant digits) is the best one can expect from this multiplication since, to begin with, one of the values had only two significant digits. Another way of looking at this is to say that beyond the first two digits in the answer, the rest of the digits are meaningless or not significant. For example, if one reports what the calculator displays, 2.3601 times 0.34 equals 0.802434, the last four digits are *meaningless*. As shown, the final result may lie between 0.79 and 0.81—any digits beyond the two significant digits are not only meaningless, but *misleading,* since they imply to the reader more precision than is really there.

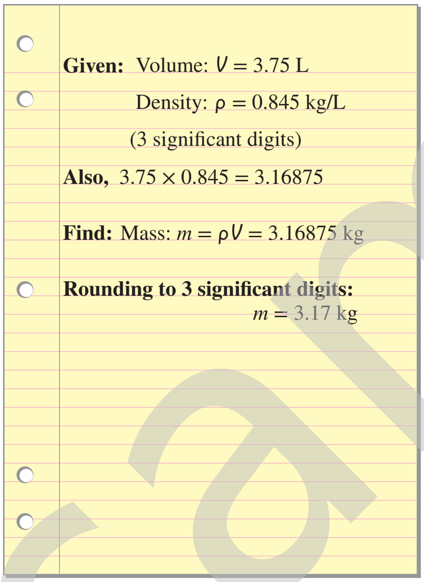


FIGURE 1–53
A result with more significant digits than that of given data falsely implies more precision.

As another example, consider a 3.75-L container filled with gasoline whose density is 0.845 kg/L, and determine its mass. Probably the first thought that comes to your mind is to multiply the volume and density to obtain 3.16875 kg for the mass, which falsely implies that the mass so determined is precise to six significant digits. In reality, however, the mass cannot be more precise than three significant digits since both the volume and the density are precise to three significant digits only. Therefore, the result should be rounded to three significant digits, and the mass should be reported to be 3.17 kg instead of what the calculator displays (Fig. 1–53). The result 3.16875 kg would be correct only if the volume and density were given to be 3.75000 L and 0.845000 kg/L, respectively. The value 3.75 L implies that we are fairly confident that the volume is precise within $\pm$0.01 L, and it cannot be 3.74 or 3.76 L. However, the volume can be 3.746, 3.750, 3.753, etc., since they all round to 3.75 L.

You should also be aware that sometimes we knowingly introduce small errors in order to avoid the trouble of searching for more accurate data. For example, when dealing with liquid water, we often use the value of 1000 kg/m^3 for density, which is the density value of pure water at 0°C. Using this value at 75°C will result in an error of 2.5 percent since the density at this temperature is 975 kg/m^3. The minerals and impurities in the water will introduce additional error. This being the case, you should have no reservation in rounding the final results to a reasonable number of significant digits. Besides, having a few percent uncertainty in the results of engineering analysis is usually the norm, not the exception.

When writing intermediate results in a computation, it is advisable to keep several "extra" digits to avoid round-off errors; however, the final result should be written with the number of significant digits taken into consideration. You must also keep in mind that a certain number of significant digits of precision in the result does not necessarily imply the same number of digits of overall *accuracy*. Bias error in one of the readings may, for example, significantly reduce the overall accuracy of the result, perhaps even rendering the last significant digit meaningless, and reducing the overall number of reliable digits by one. Experimentally determined values are subject to measurement errors, and such errors are reflected in the results obtained. For example, if the density of a substance has an uncertainty of 2 percent, then the mass determined using this density value will also have an uncertainty of 2 percent.

Finally, when the number of significant digits is unknown, the accepted engineering standard is three significant digits. Therefore, if the length of a pipe is given to be 40 m, we will assume it to be 40.0 m in order to justify using three significant digits in the final results.

EXAMPLE 1–6 Significant Digits and Volume Flow Rate

Jennifer is conducting an experiment that uses cooling water from a garden hose. In order to calculate the volume flow rate of water through the hose, she times how long it takes to fill a container (Fig. 1–54). The volume of water collected is $V = 1.1$ gal in time period $\Delta t = 45.62$ s, as measured with a stopwatch. Calculate the volume flow rate of water through the hose in units of cubic meters per minute.

SOLUTION Volume flow rate is to be determined from measurements of volume and time period.

Assumptions **1** Jennifer recorded her measurements properly, such that the volume measurement is precise to two significant digits while the time period is precise to four significant digits. **2** No water is lost due to splashing out of the container.

Analysis Volume flow rate $\dot{V}$ is volume displaced per unit time and is expressed as

Volume flow rate: $$\dot{V} = \frac{\Delta V}{\Delta t}$$

Substituting the measured values, the volume flow rate is determined to be

$$\dot{V} = \frac{1.1 \text{ gal}}{45.62 \text{ s}}\left(\frac{3.7854 \times 10^{-3} \text{ m}^3}{1 \text{ gal}}\right)\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = \mathbf{5.5 \times 10^{-3}\ m^3/min}$$

Discussion The final result is listed to two significant digits since we cannot be confident of any more precision than that. If this were an intermediate step in subsequent calculations, a few extra digits would be carried along to avoid accumulated round-off error. In such a case, the volume flow rate would be written as $\dot{V} = 5.4765 \times 10^{-3}$ m^3/min. Based on the given information, we cannot say anything about the *accuracy* of our result, since we have no information about systematic errors in either the volume measurement or the time measurement.

Also keep in mind that good precision does not guarantee good accuracy. For example, if the stopwatch had not been properly calibrated, its accuracy could be quite poor, yet the readout would still be displayed to four significant digits of precision.

In common practice, precision is often associated with *resolution,* which is a measure of how finely the instrument can report the measurement. For example, a digital voltmeter with five digits on its display is said to be more precise than a digital voltmeter with only three digits. However, the number of displayed digits has nothing to do with the overall *accuracy* of the measurement. An instrument can be very precise without being very accurate when there are significant bias errors. Likewise, an instrument with very few displayed digits can be more accurate than one with many digits (Fig. 1–55).

FIGURE 1–54
Photo for Example 1–6 for the measurement of volume flow rate.
Photo by John M. Cimbala.

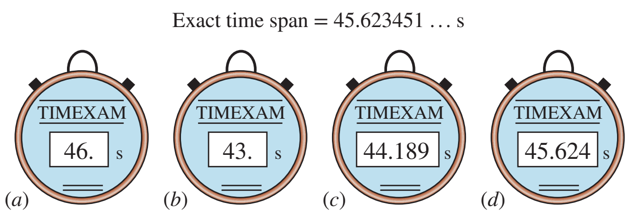
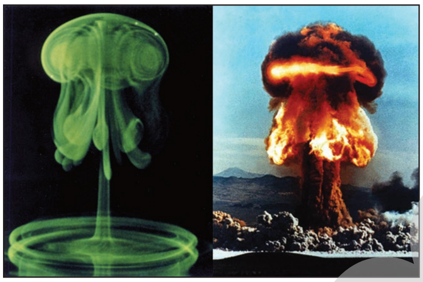


FIGURE 1–55
An instrument with many digits of resolution (stopwatch *c*) may be less accurate than an instrument with few digits of resolution (stopwatch *a*). What can you say about stopwatches *b* and *d*?

APPLICATION SPOTLIGHT ■ What Nuclear Blasts and Raindrops Have in Common

Guest Author: Lorenz Sigurdson, Vortex Fluid Dynamics Lab, University of Alberta



(*a*) (*b*)

FIGURE 1–56
Comparison of the vortex structure created by: (*a*) a water drop after impacting a pool of water (inverted, from Peck and Sigurdson, 1994), and (*b*) an above-ground nuclear test in Nevada in 1957 (U.S. Department of Energy). The 2.6 mm drop was dyed with fluorescent tracer and illuminated by a strobe flash 50 ms after it had fallen 35 mm and impacted the clear pool. The drop was approximately spherical at the time of impact with the clear pool of water. Interruption of a laser beam by the falling drop was used to trigger a timer that controlled the time of the strobe flash after impact of the drop. Details of the careful experimental procedure necessary to create the drop photograph are given by Peck and Sigurdson (1994) and Peck et al. (1995). The tracers added to the flow in the bomb case were primarily heat and dust. The heat is from the original fireball which for this particular test (the "Priscilla" event of Operation Plumbob) was large enough to reach the ground from where the bomb was initially suspended. Therefore, the tracer's initial geometric condition was a sphere intersecting the ground.

Why do the two images in Fig. 1–56 look alike? Figure 1–56*b* shows an above-ground nuclear test performed by the U.S. Department of Energy in 1957. A nuclear blast created a fireball on the order of 100 m in diameter. Expansion is so quick that a compressible flow feature occurs: an expanding spherical shock wave. The image shown in Fig. 1–56*a* is an everyday innocuous event: an *inverted* image of a dye-stained water drop after it has fallen into a pool of water, looking from below the pool surface. It could have fallen from your spoon into a cup of coffee, or been a secondary splash after a raindrop hit a lake. Why is there such a strong similarity between these two vastly different events? The application of fundamental principles of fluid mechanics learned in this book will help you understand much of the answer, although one can go much deeper.

The water has higher *density* (Chap. 2) than air, so the drop has experienced negative *buoyancy* (Chap. 3) as it has fallen through the air before impact. The fireball of hot gas is less dense than the cool air surrounding it, so it has positive buoyancy and rises. The *shock wave* (Chap. 12) reflecting from the ground also imparts a positive upward force to the fireball. The primary structure at the top of each image is called a *vortex ring*. This ring is a mini-tornado of concentrated *vorticity* (Chap. 4) with the ends of the tornado looping around to close on itself. The laws of *kinematics* (Chap. 4) tell us that this vortex ring will carry the fluid in a direction toward the top of the page. This is expected in both cases from the forces applied and the law of conservation of momentum applied through a *control volume analysis* (Chap. 5). One could also analyze this problem with *differential analysis* (Chaps. 9 and 10) or with *computational fluid dynamics* (Chap. 15). But why does the *shape* of the tracer material look so similar? This occurs if there is approximate *geometric* and *kinematic similarity* (Chap. 7), and if the *flow visualization* (Chap. 4) technique is similar. The passive tracers of heat and dust for the bomb, and fluorescent dye for the drop, were introduced in a similar manner as noted in the figure caption.

Further knowledge of kinematics and vortex dynamics can help explain the similarity of the vortex structure in the images to much greater detail, as discussed by Sigurdson (1997) and Peck and Sigurdson (1994). Look at the lobes dangling beneath the primary vortex ring, the striations in the "stalk," and the ring at the base of each structure. There is also topological similarity of this structure to other vortex structures occurring in turbulence. Comparison of the drop and bomb has given us a better understanding of how turbulent structures are created and evolve. What other secrets of fluid mechanics are left to be revealed in explaining the similarity between these two flows?

References

Peck, B., and Sigurdson, L.W., "The Three-Dimensional Vortex Structure of an Impacting Water Drop," *Phys. Fluids*, 6(2) (Part 1), p. 564, 1994.

Peck, B., Sigurdson, L.W., Faulkner, B., and Buttar, I., "An Apparatus to Study Drop-Formed Vortex Rings," *Meas. Sci. Tech.*, 6, p. 1538, 1995.

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SUMMARY

In this chapter some basic concepts of fluid mechanics are introduced and discussed. A substance in the liquid or gas phase is referred to as a *fluid*. *Fluid mechanics* is the science that deals with the behavior of fluids at rest or in motion and the interaction of fluids with solids or other fluids at the boundaries.

The flow of an unbounded fluid over a surface is *external flow,* and the flow in a pipe or duct is *internal flow* if the fluid is completely bounded by solid surfaces. A fluid flow is classified as being *compressible* or *incompressible,* depending on the density variation of the fluid during flow. The densities of liquids are essentially constant, and thus the flow of liquids is typically incompressible. The term *steady* implies *no change with time*. The opposite of steady is *unsteady*. The term *uniform* implies *no change with location* over a specified region. A flow is said to be *one-dimensional* when the properties or variables change in one dimension only. A fluid in direct contact with a solid surface sticks to the surface and there is no slip. This is known as the *no-slip condition,* which leads to the formation of *boundary layers* along solid surfaces. In this book we concentrate on steady incompressible viscous flows—both internal and external.

A system of fixed mass is called a *closed system,* and a system that involves mass transfer across its boundaries is called an *open system* or *control volume*. A large number of engineering problems involve mass flow in and out of a system and are therefore modeled as control volumes.

In engineering calculations, it is important to pay particular attention to the units of the quantities to avoid errors caused by inconsistent units, and to follow a systematic approach. It is also important to recognize that the information given is not known to more than a certain number of significant digits, and the results obtained cannot possibly be accurate to more significant digits. The information given on dimensions and units; problem-solving technique; and accuracy, precision, and significant digits will be used throughout the entire text.

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2. G. M. Homsy, H. Aref, K. S. Breuer, S. Hochgreb, J. R. Koseff, B. R. Munson, K. G. Powell, C. R. Robertson, and S. T. Thoroddsen. *Multi-Media Fluid Mechanics* (CD). Cambridge: Cambridge University Press, 2000.
3. M. Van Dyke. *An Album of Fluid Motion.* Stanford, CA: The Parabolic Press, 1982.

PROBLEMS*

Introduction, Classification, and System

1–1C What is a fluid? How does it differ from a solid? How does a gas differ from a liquid?

1–2C Define internal, external, and open-channel flows.

1–3C Define incompressible flow and incompressible fluid. Must the flow of a compressible fluid necessarily be treated as compressible?

1–4C Consider the flow of air over the wings of an aircraft. Is this flow internal or external? How about the flow of gases through a jet engine?

1–5C What is forced flow? How does it differ from natural flow? Is flow caused by winds forced or natural flow?

1–6C How is the Mach number of a flow defined? What does a Mach number of 2 indicate?

1–7C When an airplane is flying at a constant speed relative to the ground, is it correct to say that the Mach number of this airplane is also constant?

1–8C Consider the flow of air at a Mach number of 0.12. Should this flow be approximated as being incompressible?

1–9C What is the no-slip condition? What causes it?

1–10C What is a boundary layer? What causes a boundary layer to develop?

1–11C What is a steady-flow process?

1–12C Define stress, normal stress, shear stress, and pressure.

1–13C What are system, surroundings, and boundary?

* Problems designated by a “C” are concept questions, and students are encouraged to answer them all. Problems designated by an “E” are in English units, and SI users can ignore them. Problems with the icon are comprehensive in nature and are intended to be solved with appropriate software.

1–14C When analyzing the acceleration of gases as they flow through a nozzle, what would you choose as your system? What type of system is this?

1–15C When is a system a closed system, and when is it a control volume?

1–16C You are trying to understand how a reciprocating air compressor (a piston-cylinder device) works. What system would you use? What type of system is this?

Mass, Force, and Units

1–17C What is the difference between pound-mass and pound-force?

1–18C In a news article, it is stated that a recently developed geared turbofan engine produces 15,000 pounds of thrust to propel the aircraft forward. Is "pound" mentioned here lbm or lbf? Explain.

1–19C Explain why the light-year has the dimension of length.

1–20C What is the net force acting on a car cruising at a constant velocity of 70 km/h (*a*) on a level road and (*b*) on an uphill road?

1–21 A man goes to a traditional market to buy a steak for dinner. He finds a 12-oz steak (1 lbm = 16 oz) for \$3.15. He then goes to the adjacent international market and finds a 320-g steak of identical quality for \$3.30. Which steak is the better buy?

1–22 What is the weight, in N, of an object with a mass of 150 kg at a location where $g = 9.6\ \text{m/s}^2$?

1–23 What is the weight of a 1-kg substance in N, kN, kg·m/s^2, kgf, lbm·ft/s^2, and lbf?

1–24 Determine the mass and the weight of the air contained in a room whose dimensions are 3 m × 5 m × 7 m. Assume the density of the air is $1.16\ \text{kg/m}^3$. *Answers:* 122 kg, 1195 N

1–25 A 3-kW resistance heater in a water heater runs for 2 hours to raise the water temperature to the desired level. Determine the amount of electric energy used in both kWh and kJ.

1–26E A 195-lbm astronaut took his bathroom scale (a spring scale) and a beam scale (compares masses) to the moon where the local gravity is $g = 5.48\ \text{ft/s}^2$. Determine how much he will weigh (*a*) on the spring scale and (*b*) on the beam scale. *Answers:* (*a*) 33.2 lbf, (*b*) 195 lbf

1–27 The acceleration of high-speed aircraft is sometimes expressed in *g*'s (in multiples of the standard acceleration of gravity). Determine the net force, in N, that a 90-kg man would experience in an aircraft whose acceleration is 6 *g*'s.

1–28 A 10-kg rock is thrown upward with a force of 280 N at a location where the local gravitational acceleration is $9.79\ \text{m/s}^2$. Determine the acceleration of the rock, in m/s^2.

1–29 Solve Prob. 1–30 using appropriate software. Print out the entire solution, including the numerical results with proper units.

1–30 The value of the gravitational acceleration *g* decreases with elevation from $9.807\ \text{m/s}^2$ at sea level to $9.767\ \text{m/s}^2$ at an altitude of 13,000 m, where large passenger planes cruise. Determine the percent reduction in the weight of an airplane cruising at 13,000 m relative to its weight at sea level.

1–31 At 45° latitude, the gravitational acceleration as a function of elevation z above sea level is given by $g = a - bz$, where $a = 9.807\ \text{m/s}^2$ and $b = 3.32 \times 10^{-6}\ \text{s}^{-2}$. Determine the height above sea level where the weight of an object will decrease by 1 percent. *Answer:* 29,500 m

1–32 The gravitational constant *g* is $9.807\ \text{m/s}^2$ at sea level, but it decreases as you go up in elevation. A useful equation for this decrease in *g* is $g = a - bz$, where z is the elevation above sea level, $a = 9.807\ \text{m/s}^2$, and $b = 3.32 \times 10^{-6}\ 1/\text{s}^2$. An astronaut "weighs" 80.0 kg at sea level. [Technically this means that his/her mass is 80.0 kg.] Calculate this person's weight in N while floating around in the International Space Station ($z = 354$ km). If the Space Station were to suddenly stop in its orbit, what gravitational acceleration would the astronaut feel immediately after the satellite stopped moving? In light of your answer, explain why astronauts on the Space Station feel "weightless."

1–33 On average, an adult person breathes in about 7.0 liters of air per minute. Assuming atmospheric pressure and 20°C air temperature, estimate the mass of air in kilograms that a person breathes in per day.

1–34 While solving a problem, a person ends up with the equation $E = 16\ \text{kJ} + 7\ \text{kJ/kg}$ at some stage. Here E is the total energy and has the unit of kilojoules. Determine how to correct the error and discuss what may have caused it.

1–35 An airplane flies horizontally at 70 m/s. Its propeller delivers 1500 N of thrust (forward force) to overcome aerodynamic drag (backward force). Using dimensional reasoning and unity converstion ratios, calculate the useful power delivered by the propeller in units of kW and horsepower.

1–36 If the airplane of Prob. 1–35 weighs 1700 lbf, estimate the lift force produced by the airplane's wings (in lbf and newtons) when flying at 70.0 m/s.

1–37E The boom of a fire truck raises a fireman (and his equipment—total weight 280 lbf) 40 ft into the air to fight a building fire. (*a*) Showing all your work and using unity conversion ratios, calculate the work done by the boom on the fireman in units of Btu. (*b*) If the useful power supplied by the boom to lift the fireman is 3.50 hp, estimate how long it takes to lift the fireman.

1–38 A 6-kg plastic tank that has a volume of $0.18\ \text{m}^3$ is filled with liquid water. Assuming the density of water is $1000\ \text{kg/m}^3$, determine the weight of the combined system.

1–39 Water at 15°C from a garden hose fills a 1.5 L container in 2.85 s. Using unity conversion ratios and showing all your work, calculate the volume flow rate in liters per minute (Lpm) and the mass flow rate in kg/s.

1–40 A forklift raises a 90.5 kg crate 1.80 m. (*a*) Showing all your work and using unity conversion ratios, calculate the work done by the forklift on the crane, in units of kJ. (*b*) If it takes 12.3 seconds to lift the crate, calculate the useful power supplied to the crate in kilowatts.

1–41 The gas tank of a car is filled with a nozzle that discharges gasoline at a constant flow rate. Based on unit considerations of quantities, obtain a relation for the filling time in terms of the volume V of the tank (in L) and the discharge rate of gasoline ($\dot{V}$, in L/s).

1–42 A pool of volume V (in m^3) is to be filled with water using a hose of diameter D (in m). If the average discharge velocity is V (in m/s) and the filling time is t (in s), obtain a relation for the volume of the pool based on unit considerations of quantities involved.

1–43 Based on unit considerations alone, show that the power needed to accelerate a car of mass m (in kg) from rest to velocity V (in m/s) in time interval t (in s) is proportional to mass and the square of the velocity of the car and inversely proportional to the time interval.

Modeling and Solving Engineering Problems

1–44C What is the importance of modeling in engineering? How are the mathematical models for engineering processes prepared?

1–45C What is the difference between the analytical and experimental approach to engineering problems? Discuss the advantages and disadvantages of each approach.

1–46C When modeling an engineering process, how is the right choice made between a simple but crude and a complex but accurate model? Is the complex model necessarily a better choice since it is more accurate?

1–47C What is the difference between precision and accuracy? Can a measurement be very precise but inaccurate? Explain.

1–48C How do the differential equations in the study of a physical problem arise?

1–49C What is the value of the engineering software packages in (*a*) engineering education and (*b*) engineering practice?

1–50 Solve this system of three equations with three unknowns using appropriate software:

$$2x - y + z = 9$$
$$3x^2 + 2y = z + 2$$
$$xy + 2z = 14$$

1–51 Solve this system of two equations with two unknowns using appropriate software:

$$x^3 - y^2 = 10.5$$
$$3xy + y = 4.6$$

1–52 Determine a positive real root of this equation using appropriate software:

$$3.5x^3 - 10x^{0.5} - 3x = -4$$

1–53 Solve this system of three equations with three unknowns using appropriate software:

$$x^2y - z = 1.5$$
$$x - 3y^{0.5} + xz = -2$$
$$x + y - z = 4.2$$

Review Problems

1–54E A student buys a 5000 Btu window air conditioner for his apartment bedroom. He monitors it for one hour on a hot day and determines that it operates approximately 60 percent of the time (duty cycle = 60 percent) to keep the room at nearly constant temperature. (*a*) Showing all your work and using unity conversion ratios, calculate the rate of heat transfer into the bedroom through the walls, windows, etc. in units of Btu/h and in units of kW. (*b*) If the energy efficiency ratio (EER) of the air conditioner is 9.0 and electricity costs 7.5 cents per kilowatt-hr, calculate how much it costs (in cents) for him to run the air conditioner for one hour.

1–55 The weight of bodies may change somewhat from one location to another as a result of the variation of the gravitational acceleration g with elevation. Accounting for this variation using the relation in Prob. 1–31, determine the weight of an 65-kg person at sea level ($z = 0$), in Denver ($z = 1610$ m), and on the top of Mount Everest ($z = 8848$ m).

1–56E The reactive force developed by a jet engine to push an airplane forward is called thrust, and the thrust developed by the engine of a Boeing 777 is about 85,000 lbf. Express this thrust in N and kgf.

1–57 For liquids, the dynamic viscosity μ, which is a measure of resistance against flow is approximated as $\mu = a10^{b/(T-c)}$, where T is the absolute temperature, and a, b and c are experimental constants. Using the data listed in Table A–7 for methanol at 20ºC, 40ºC and 60ºC, determine the constant a, b and c.

1–58 An important design consideration in two-phase pipe flow of solid-liquid mixtures is the terminal settling velocity below, which the flow becomes unstable and eventually the pipe becomes clogged. On the basis of extended transportation tests, the terminal settling velocity of a solid particle in the rest water given by $V_L = F_L\sqrt{2gD(S-1)}$, where F_L is an experimental coefficient, g the gravitational acceleration, D the pipe diameter, and S the specific gravity of solid particle. What is the dimension of F_L? Is this equation dimensionally homogeneous?

1–59 Consider the flow of air through a wind turbine whose blades sweep an area of diameter D (in m). The average air velocity through the swept area is V (in m/s). On the bases of the units of the quantities involved, show that the mass flow rate of air (in kg/s) through the swept area is proportional to air density, the wind velocity, and the square of the diameter of the swept area.

1–60 A tank is filled with oil whose density is $\rho = 850$ kg/m^3. If the volume of the tank is $V = 2$ m^3, determine the amount of mass m in the tank.

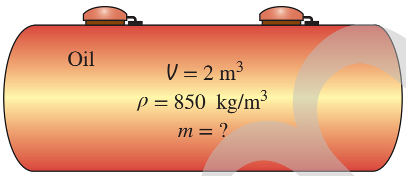


FIGURE P1–60

Fundamentals of Engineering (FE) Exam Problems

1–61 If mass, heat, and work are not allowed to cross the boundaries of a system, the system is called
(*a*) Isolated (*b*) Isothermal (*c*) Adiabatic (*d*) Control mass (*e*) Control volume

1–62 The speed of an aircraft is given to be 260 m/s in air. If the speed of sound at that location is 330 m/s, the flight of aircraft is
(*a*) Sonic (*b*) Subsonic (*c*) Supersonic (*d*) Hypersonic

1–63 One J/kg is equal to
(*a*) 1 kPa·m^3 (*b*) 1 kN·m/kg (*c*) 0.001 kJ
(*d*) 1 N·m (*e*) 1 m^2/s^2

1–64 Which is a unit for power?
(*a*) Btu (*b*) kWh (*c*) kcal (*d*) hph (*e*) kW

1–65 The speed of an aircraft is given to be 950 km/h. If the speed of sound at that location is 315 m/s, the Mach number is
(*a*) 0.63 (*b*) 0.84 (*c*) 1.0 (*d*) 1.07 (*e*) 1.20

1–66 The weight of a 10-kg mass at sea level is
(*a*) 9.81 N (*b*) 32.2 kgf (*c*) 98.1 N (*d*) 10 N (*e*) 100 N

1–67 The weight of a 1-lbm mass is
(*a*) 1 lbm·ft/s^2 (*b*) 9.81 lbf (*c*) 9.81 N (*d*) 32.2 lbf (*e*) 1 lbf

1–68 A hydroelectric power plant operates at its rated power of 12 MW. If the plant has produced 26 million kWh of electricity in a specified year, the number of hours the plant has operated that year is
(*a*) 2167 h (*b*) 2508 h (*c*) 3086 h (*d*) 3710 h (*e*) 8760 h

Design and Essay Problems

1–69 Write an essay on the various mass- and volume-measurement devices used throughout history. Also, explain the development of the modern units for mass and volume.

1–70 Search the Internet to find out how to properly add or subtract numbers while taking into consideration the number of significant digits. Write a summary of the proper technique, then use the technique to solve the following cases: (*a*) 1.006 + 23.47, (*b*) 703,200 − 80.4, and (*c*) 4.6903 − 14.58. Be careful to express your final answer to the appropriate number of significant digits.

1–71 Another unit is kgf, which is a force unit used mostly in Europe, and is defined as kp (kilopond). Explain the difference between kilopond (kp = kgf) and kilopound (10^3 lbf) from force units and write the unity conversion factor between them. The density of water at 4°C is = 1000 kg/m^3. Express this density value in units of kp·m^{-4}·s^2.

1–72 Discuss why pressure tests of pressurized tanks such as steam boilers, pipes, and tanks including gases such as nitrogen, air, oxygen, etc. with high pressure are carried out hydrostatically by using liquids such as water and hydraulic oil.

CHAPTER

2

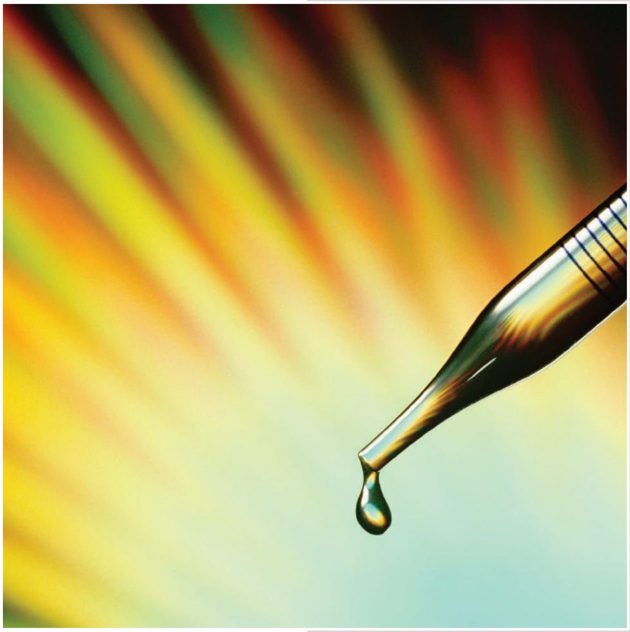
PROPERTIES OF FLUIDS

In this chapter, we discuss properties that are encountered in the analysis of fluid flow. First we discuss *intensive* and *extensive properties* and define *density* and *specific gravity*. This is followed by a discussion of the properties *vapor pressure*, *energy* and its various forms, the *specific heats* of ideal gases and incompressible substances, the *coefficient of compressibility,* and the *speed of sound*. Then we discuss the property *viscosity*, which plays a dominant role in most aspects of fluid flow. Finally, we present the property *surface tension* and determine the *capillary rise* from static equilibrium conditions. The property *pressure* is discussed in Chap. 3 together with fluid statics.

OBJECTIVES

When you finish reading this chapter, you should be able to

- Have a working knowledge of the basic properties of fluids and understand the continuum approximation
- Have a working knowledge of viscosity and the consequences of the frictional effects it causes in fluid flow
- Calculate the capillary rise (or drop) in tubes due to the surface tension effect



A drop forms when liquid is forced out of a small tube. The shape of the drop is determined by a balance of pressure, gravity, and surface tension forces.

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2–1 ▪ INTRODUCTION

Any characteristic of a system is called a **property**. Some familiar properties are pressure P, temperature T, volume V, and mass m. The list can be extended to include less familiar ones such as viscosity, thermal conductivity, modulus of elasticity, thermal expansion coefficient, electric resistivity, and even velocity and elevation.

Properties are considered to be either *intensive* or *extensive*. **Intensive properties** are those that are independent of the mass of the system, such as temperature, pressure, and density. **Extensive properties** are those whose values depend on the size—or extent—of the system. Total mass, total volume V, and total momentum are some examples of extensive properties. An easy way to determine whether a property is intensive or extensive is to divide the system into two equal parts with an imaginary partition, as shown in Fig. 2–1. Each part will have the same value of intensive properties as the original system, but half the value of the extensive properties.

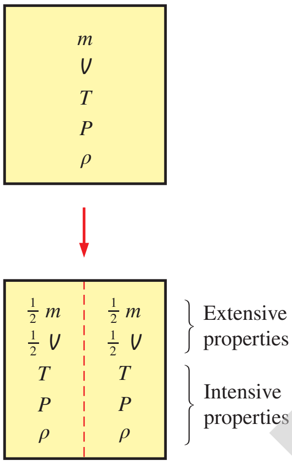

FIGURE 2–1
Criterion to differentiate intensive and extensive properties.

Generally, uppercase letters are used to denote extensive properties (with mass m being a major exception), and lowercase letters are used for intensive properties (with pressure P and temperature T being the obvious exceptions).

Extensive properties per unit mass are called **specific properties**. Some examples of specific properties are specific volume ($v = V/m$) and specific total energy ($e = E/m$).

The state of a system is described by its properties. But we know from experience that we do not need to specify all the properties in order to fix a state. Once the values of a sufficient number of properties are specified, the rest of the properties assume certain values. That is, specifying a certain number of properties is sufficient to fix a state. The number of properties required to fix the state of a system is given by the **state postulate**: *The state of a simple compressible system is completely specified by two independent, intensive properties.*

Two properties are independent if one property can be varied while the other one is held constant. Not all properties are independent, and some are defined in terms of others, as explained in Section 2–2.

Continuum

A fluid is composed of molecules which may be widely spaced apart, especially in the gas phase. Yet it is convenient to disregard the atomic nature of the fluid and view it as continuous, homogeneous matter with no holes, that is, a **continuum**. The continuum idealization allows us to treat properties as point functions and to assume that the properties vary continually in space with no jump discontinuities. This idealization is valid as long as the size of the system we deal with is large relative to the space between the molecules (Fig. 2–2). This is the case in practically all problems, except some specialized ones. The continuum idealization is implicit in many statements we make, such as “the density of water in a glass is the same at any point.”

FIGURE 2–2
The length scale associated with most flows, such as seagulls in flight, is orders of magnitude larger than the mean free path of the air molecules. Therefore, here, and for all fluid flows considered in this book, the continuum idealization is appropriate.
© PhotoLink/Getty Images RF

To have a sense of the distances involved at the molecular level, consider a container filled with oxygen at atmospheric conditions. The diameter of an oxygen molecule is about 3×10^{-10} m and its mass is 5.3×10^{-26} kg. Also, the *mean free path* λ of oxygen at 1 atm pressure and 20°C is 6.3×10^{-8} m. That is, an oxygen molecule travels, on average, a distance of 6.3×10^{-8} m (about 200 times its diameter) before it collides with another molecule.

Also, there are about 3×10^{16} molecules of oxygen in the tiny volume of 1 mm^3 at 1 atm pressure and 20°C (Fig. 2–3). The continuum model is applicable as long as the characteristic length of the system (such as its diameter) is much larger than the mean free path of the molecules. At very low pressure, e.g., at very high elevations, the mean free path may become large (for example, it is about 0.1 m for atmospheric air at an elevation of 100 km). For such cases the **rarefied gas flow theory** should be used, and the impact of individual molecules should be considered. In this text we limit our consideration to substances that can be modeled as a continuum. Quantitatively, a dimensionless number called the Knudsen number Kn = λ/L is defined, where λ is the mean free path of the fluid molecules and L is some characteristic length scale of the fluid flow. If Kn is very small (typically less than about 0.01), the fluid medium can be approximated as a continuum medium.

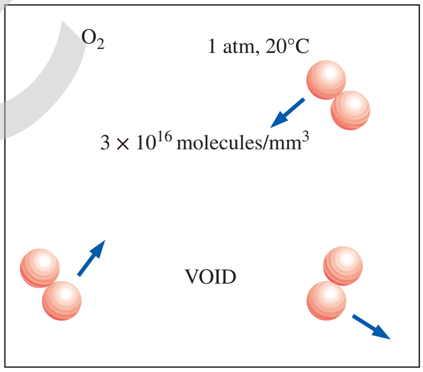

FIGURE 2–3
Despite the relatively large gaps between molecules, a gas can usually be treated as a continuum because of the very large number of molecules even in an extremely small volume.

2–2 ▪ DENSITY AND SPECIFIC GRAVITY

Density is defined as *mass per unit volume* (Fig. 2–4). That is,

Density:
$$\rho = \frac{m}{V} \qquad (\text{kg/m}^3) \tag{2–1}$$

The reciprocal of density is the **specific volume** υ, which is defined as *volume per unit mass*. That is, $\upsilon = V/m = 1/\rho$. For a differential volume element of mass δm and volume δV, density can be expressed as $\rho = \delta m/\delta V$.

The density of a substance, in general, depends on temperature and pressure. The density of most gases is proportional to pressure and inversely proportional to temperature. Liquids and solids, on the other hand, are essentially incompressible substances, and the variation of their density with pressure is usually negligible. At 20°C, for example, the density of water changes from 998 kg/m^3 at 1 atm to 1003 kg/m^3 at 100 atm, a change of just 0.5 percent. The density of liquids and solids depends more strongly on temperature than it does on pressure. At 1 atm, for example, the density of water changes from 998 kg/m^3 at 20°C to 975 kg/m^3 at 75°C, a change of 2.3 percent, which can still be neglected in many engineering analyses.

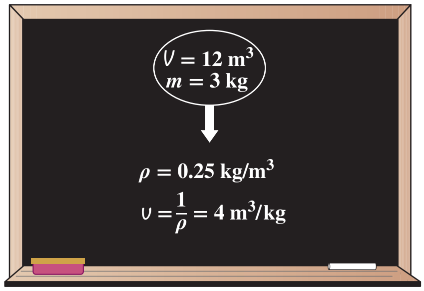

FIGURE 2–4
Density is mass per unit volume; specific volume is volume per unit mass.

Sometimes the density of a substance is given relative to the density of a well-known substance. Then it is called **specific gravity**, or **relative density**, and is defined as *the ratio of the density of a substance to the density of some standard substance at a specified temperature* (usually water at 4°C, for which $\rho_{H_2O} = 1000$ kg/m^3). That is,

Specific gravity:
$$SG = \frac{\rho}{\rho_{H_2O}} \tag{2–2}$$

Note that the specific gravity of a substance is a dimensionless quantity. However, in SI units, the numerical value of the specific gravity of a substance is exactly equal to its density in g/cm^3 or kg/L (or 0.001 times the density in kg/m^3) since the density of water at 4°C is 1 g/cm^3 = 1 kg/L = 1000 kg/m^3. The specific gravity of mercury at 20°C, for example, is 13.6. Therefore, its density at 20°C is 13.6 g/cm^3 = 13.6 kg/L = 13,600 kg/m^3. The specific gravities of some substances at 20°C are given in Table 2–1. Note that substances with specific gravities less than 1 are lighter than water, and thus they would float on water (if immiscible).

TABLE 2–1
The specific gravity of some substances at 20°C and 1 atm unless stated otherwise

Substance	SG
Water	1.0
Blood (at 37°C)	1.06
Seawater	1.025
Gasoline	0.68
Ethyl alcohol	0.790
Mercury	13.6
Balsa wood	0.17
Dense oak wood	0.93
Gold	19.3
Bones	1.7–2.0
Ice (at 0°C)	0.916
Air	0.001204

The weight of a unit volume of a substance is called **specific weight** or **weight density** and is expressed as

$$\textit{Specific weight:} \qquad \gamma_s = \rho g \qquad (\mathrm{N/m^3}) \qquad \textbf{(2–3)}$$

where g is the gravitational acceleration.

Recall from Chap. 1 that the densities of liquids are essentially constant, and thus they can often be approximated as being incompressible substances during most processes without sacrificing much in accuracy.

Density of Ideal Gases

Property tables provide very accurate and precise information about the properties, but sometimes it is convenient to have some simple relations among the properties that are sufficiently general and reasonably accurate. Any equation that relates the pressure, temperature, and density (or specific volume) of a substance is called an **equation of state**. The simplest and best-known equation of state for substances in the gas phase is the **ideal-gas equation of state**, expressed as

$$P\upsilon = RT \quad \text{or} \quad P = \rho RT \qquad \textbf{(2–4)}$$

where P is the absolute pressure, υ is the specific volume, T is the thermodynamic (absolute) temperature, ρ is the density, and R is the gas constant. The gas constant R is different for each gas and is determined from $R = R_u/M$, where R_u is the **universal gas constant** whose value is $R_u = 8.314$ kJ/kmol·K = 1.986 Btu/lbmol·R, and M is the *molar mass* (also called *molecular weight*) of the gas. The values of R and M for several substances are given in Table A–1.

The thermodynamic temperature scale in the SI is the **Kelvin scale**, and the temperature unit on this scale is the **kelvin**, designated by K. In the English system, it is the **Rankine scale**, and the temperature unit on this scale is the **rankine**, R. Various temperature scales are related to each other by

$$T(\mathrm{K}) = T(^\circ\mathrm{C}) + 273.15 = T(\mathrm{R})/1.8 \qquad \textbf{(2–5)}$$

$$T(\mathrm{R}) = T(^\circ\mathrm{F}) + 459.67 = 1.8\,T(\mathrm{K}) \qquad \textbf{(2–6)}$$

It is common practice to round the constants 273.15 and 459.67 to 273 and 460, respectively, but we do not encourage this practice.

Equation 2–4, the ideal-gas equation of state, is also called simply the **ideal-gas relation**, and a gas that obeys this relation is called an **ideal gas**. For an ideal gas of volume V, mass m, and number of moles $N = m/M$, the ideal-gas equation of state can also be written as $PV = mRT$ or $PV = NR_uT$. For a fixed mass m, writing the ideal-gas relation twice and simplifying, the properties of an ideal gas at two different states are related to each other by $P_1V_1/T_1 = P_2V_2/T_2$.

An ideal gas is a hypothetical substance that obeys the relation $P\upsilon = RT$. It has been experimentally observed that the ideal-gas relation closely approximates the P-υ-T behavior of real gases at low densities. At low pressures and high temperatures, the density of a gas decreases and the gas behaves like an ideal gas (Fig. 2–5). In the range of practical interest, many familiar

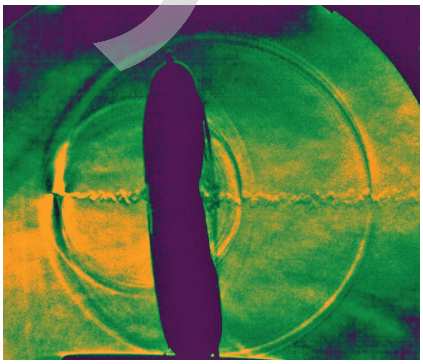

FIGURE 2–5
Air behaves as an ideal gas, even at very high speeds. In this schlieren image, a bullet traveling at about the speed of sound bursts through both sides of a balloon, forming two expanding shock waves. The turbulent wake of the bullet is also visible.

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gases such as air, nitrogen, oxygen, hydrogen, helium, argon, neon, and carbon dioxide and even heavier gases such as krypton can be treated as ideal gases with negligible error (often less than 1 percent). Dense gases such as water vapor in steam power plants and refrigerant vapor in refrigerators, air conditioners, and heat pumps, however, should not be treated as ideal gases since they usually exist at a state near saturation.

EXAMPLE 2–1 Density, Specific Gravity, and Mass of Air in a Room

Determine the density, specific gravity, and mass of the air in a room whose dimensions are 4 m × 5 m × 6 m at 100 kPa and 25°C (Fig. 2–6).

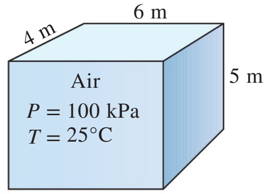


FIGURE 2–6
Schematic for Example 2–1.

SOLUTION The density, specific gravity, and mass of the air in a room are to be determined.

Assumptions At specified conditions, air can be treated as an ideal gas.

Properties The gas constant of air is $R = 0.287\ \text{kPa·m}^3/\text{kg·K}$.

Analysis The density of the air is determined from the ideal-gas relation $P = \rho RT$ to be

$$\rho = \frac{P}{RT} = \frac{100\ \text{kPa}}{(0.287\ \text{kPa·m}^3/\text{kg·K})(25 + 273.15)\ \text{K}} = \mathbf{1.17\ kg/m^3}$$

Then the specific gravity of the air becomes

$$\text{SG} = \frac{\rho}{\rho_{H_2O}} = \frac{1.17\ \text{kg/m}^3}{1000\ \text{kg/m}^3} = \mathbf{0.00117}$$

Finally, the volume and the mass of the air in the room are

$$V = (4\ \text{m})(5\ \text{m})(6\ \text{m}) = 120\ \text{m}^3$$

$$m = \rho V = (1.17\ \text{kg/m}^3)(120\ \text{m}^3) = \mathbf{140\ kg}$$

Discussion Note that we converted the temperature to (absolute) unit K from (relative) unit °C before using it in the ideal-gas relation.

2–3 ▪ VAPOR PRESSURE AND CAVITATION

It is well-established that temperature and pressure are dependent properties for pure substances during phase-change processes, and there is one-to-one correspondence between temperature and pressure. At a given pressure, the temperature at which a pure substance changes phase is called the **saturation temperature** T_{sat}. Likewise, at a given temperature, the pressure at which a pure substance changes phase is called the **saturation pressure** P_{sat}. At an absolute pressure of 1 standard atmosphere (1 atm or 101.325 kPa), for example, the saturation temperature of water is 100°C. Conversely, at a temperature of 100°C, the saturation pressure of water is 1 atm.

The **vapor pressure** P_v of a pure substance is defined as *the pressure exerted by its vapor in phase equilibrium with its liquid at a given temperature* (Fig. 2–7). P_v is a property of the pure substance, and turns out to be identical to the saturation pressure P_{sat} of the liquid ($P_v = P_{sat}$). We must be

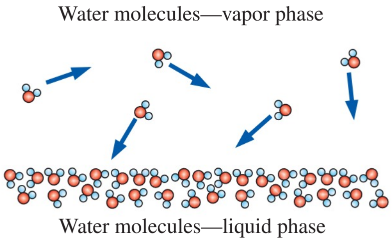


FIGURE 2–7
The vapor pressure (saturation pressure) of a pure substance (e.g., water) is the pressure exerted by its vapor molecules when the system is in phase equilibrium with its liquid molecules at a given temperature.

TABLE 2–2

Saturation (or vapor) pressure of water at various temperatures

Temperature T, °C	Saturation Pressure P_{sat}, kPa
−10	0.260
−5	0.403
0	0.611
5	0.872
10	1.23
15	1.71
20	2.34
25	3.17
30	4.25
40	7.38
50	12.35
100	101.3 (1 atm)
150	475.8
200	1554
250	3973
300	8581

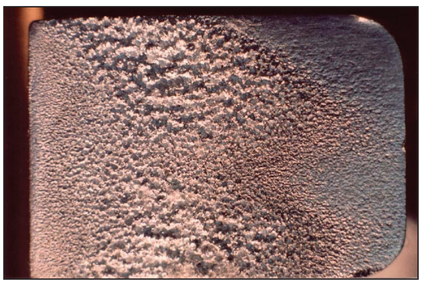


FIGURE 2–8
Cavitation damage on a 16-mm by 23-mm aluminum sample tested at 60 m/s for 2.5 hours. The sample was located at the cavity collapse region downstream of a cavity generator specifically designed to produce high damage potential.

Photograph by David Stinebring, ARL/Pennsylvania State University. Used by permission.

careful not to confuse vapor pressure with *partial pressure*. **Partial pressure** is defined as *the pressure of a gas or vapor in a mixture with other gases*. For example, atmospheric air is a mixture of dry air and water vapor, and atmospheric pressure is the sum of the partial pressure of dry air and the partial pressure of water vapor. The partial pressure of water vapor constitutes a small fraction (usually under 3 percent) of the atmospheric pressure since air is mostly nitrogen and oxygen. The partial pressure of a vapor must be less than or equal to the vapor pressure if there is no liquid present. However, when both vapor and liquid are present and the system is in phase equilibrium, the partial pressure of the vapor must equal the vapor pressure, and the system is said to be *saturated*. The rate of evaporation from open water bodies such as lakes is controlled by the difference between the vapor pressure and the partial pressure. For example, the vapor pressure of water at 20°C is 2.34 kPa. Therefore, a bucket of water at 20°C left in a room with dry air at 1 atm will continue evaporating until one of two things happens: the water evaporates away (there is not enough water to establish phase equilibrium in the room), or the evaporation stops when the partial pressure of the water vapor in the room rises to 2.34 kPa at which point phase equilibrium is established.

For phase-change processes between the liquid and vapor phases of a pure substance, the saturation pressure and the vapor pressure are equivalent since the vapor is pure. Note that the pressure value would be the same whether it is measured in the vapor or liquid phase (provided that it is measured at a location close to the liquid–vapor interface to avoid any hydrostatic effects). Vapor pressure increases with temperature. Thus, a substance at higher pressure boils at higher temperature. For example, water boils at 134°C in a pressure cooker operating at 3 atm absolute pressure, but it boils at 93°C in an ordinary pan at a 2000-m elevation, where the atmospheric pressure is 0.8 atm. The saturation (or vapor) pressures are given in Appendices 1 and 2 for various substances. An abridged table for water is given in Table 2–2 for easy reference.

The reason for our interest in vapor pressure is the possibility of the liquid pressure in liquid-flow systems dropping below the vapor pressure at some locations, and the resulting unplanned vaporization. For example, water at 10°C may vaporize and form bubbles at locations (such as the tip regions of impellers or suction sides of pumps) where the pressure drops below 1.23 kPa. The vapor bubbles (called **cavitation bubbles** since they form "cavities" in the liquid) collapse as they are swept away from the low-pressure regions, generating highly destructive, extremely high-pressure waves. This phenomenon, which is a common cause for drop in performance and even the erosion of impeller blades, is called **cavitation**, and it is an important consideration in the design of hydraulic turbines and pumps.

Cavitation must be avoided (or at least minimized) in most flow systems since it reduces performance, generates annoying vibrations and noise, and causes damage to equipment. We note that some flow systems use cavitation to their *advantage,* e.g., high-speed "supercavitating" torpedoes. The pressure spikes resulting from the large number of bubbles collapsing near a solid surface over a long period of time may cause erosion, surface pitting, fatigue failure, and the eventual destruction of the components or machinery (Fig. 2–8). The presence of cavitation in a flow system can be sensed by its characteristic tumbling sound.

EXAMPLE 2–2 Danger of Cavitation in a Propeller

The analysis of a propeller that operates in water at 20°C shows that the pressure at the tips of the propeller drops to 2 kPa at high speeds. Determine if there is a danger of cavitation for this propeller.

SOLUTION The minimum pressure in a propeller is given. It is to be determined if there is a danger of cavitation.

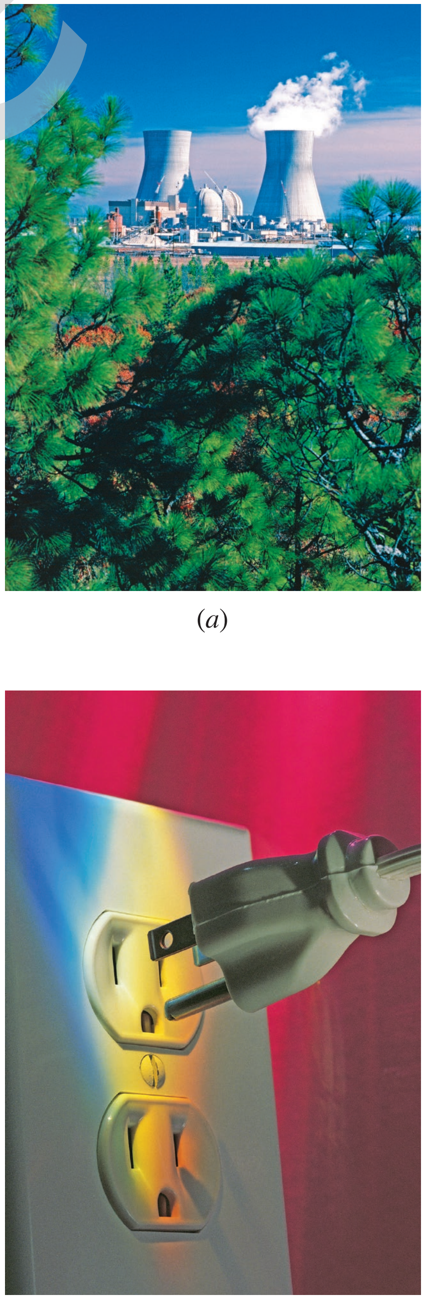
Properties The vapor pressure of water at 20°C is 2.34 kPa (Table 2–2).

Analysis To avoid cavitation, the pressure everywhere in the flow should remain above the vapor (or saturation) pressure at the given temperature, which is

$$P_v = P_{\text{sat@20°C}} = 2.34 \text{ kPa}$$

The pressure at the tip of the propeller is 2 kPa, which is less than the vapor pressure. Therefore, **there is a danger of cavitation for this propeller**.

Discussion Note that the vapor pressure increases with increasing temperature, and thus there is a greater danger of cavitation at higher fluid temperatures.



(*a*)

(*b*)

FIGURE 2–9
At least six different forms of energy are encountered in bringing power from a nuclear plant to your home, nuclear, thermal, mechanical, kinetic, magnetic, and electrical.
(a) © Creatas/PunchStock RF
(b) Comstock Images/Jupiterimages RF

2–4 ▪ ENERGY AND SPECIFIC HEATS

Energy can exist in numerous forms such as thermal, mechanical, kinetic, potential, electrical, magnetic, chemical, and nuclear (Fig. 2–9) and their sum constitutes the **total energy** E (or e on a unit mass basis) of a system. The forms of energy related to the molecular structure of a system and the degree of the molecular activity are referred to as the *microscopic energy*. The sum of all microscopic forms of energy is called the **internal energy** of a system, and is denoted by U (or u on a unit mass basis).

The *macroscopic* energy of a system is related to motion and the influence of some external effects such as gravity, magnetism, electricity, and surface tension. The energy that a system possesses as a result of its motion is called **kinetic energy**. When all parts of a system move with the same velocity, the kinetic energy per unit mass is expressed as ke $= V^2/2$ where V denotes the velocity of the system relative to some fixed reference frame. The energy that a system possesses as a result of its elevation in a gravitational field is called **potential energy** and is expressed on a per-unit mass basis as pe $= gz$ where g is the gravitational acceleration and z is the elevation of the center of gravity of the system relative to some arbitrarily selected reference plane.

In daily life, we frequently refer to the sensible and latent forms of internal energy as **heat**, and we talk about the heat content of bodies. In engineering, however, those forms of energy are usually referred to as **thermal energy** to prevent any confusion with *heat transfer*.

The international unit of energy is the *joule* (J) or *kilojoule* (1 kJ = 1000 J). A joule is 1 N times 1 m. In the English system, the unit of energy is the *British thermal unit* (Btu), which is defined as the energy needed to raise the temperature of 1 lbm of water at 68°F by 1°F. The magnitudes of kJ and Btu are almost identical (1 Btu = 1.0551 kJ). Another well-known unit of energy is the *calorie* (1 cal = 4.1868 J), which is defined as the energy needed to raise the temperature of 1 g of water at 14.5°C by 1°C.

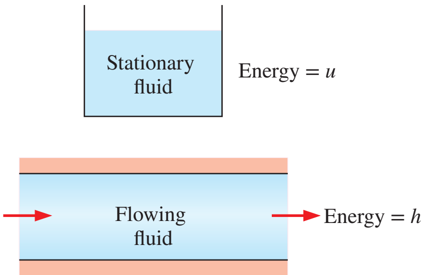


FIGURE 2–10
The *internal energy u* represents the microscopic energy of a nonflowing fluid per unit mass, whereas *enthalpy h* represents the microscopic energy of a flowing fluid per unit mass.

In the analysis of systems that involve fluid flow, we frequently encounter the combination of properties u and $P\upsilon$. For convenience, this combination is called **enthalpy** h. That is,

Enthalpy:
$$h = u + P\upsilon = u + \frac{P}{\rho} \tag{2–7}$$

where P/ρ is the *flow energy,* also called the *flow work,* which is the energy per unit mass needed to move the fluid and maintain flow. In the energy analysis of flowing fluids, it is convenient to treat the flow energy as part of the energy of the fluid and to represent the microscopic energy of a fluid stream by enthalpy h (Fig. 2–10). Note that enthalpy is a quantity per unit mass, and thus it is a *specific* property.

In the absence of such effects as magnetic, electric, and surface tension, a system is called a simple compressible system. The total energy of a simple compressible system consists of three parts: internal, kinetic, and potential energies. On a unit-mass basis, it is expressed as $e = u + \mathrm{ke} + \mathrm{pe}$. The fluid entering or leaving a control volume possesses an additional form of energy—the *flow energy* P/ρ. Then the total energy of a **flowing fluid** on a unit-mass basis becomes

$$e_{\text{flowing}} = P/\rho + e = h + \mathrm{ke} + \mathrm{pe} = h + \frac{V^2}{2} + gz \qquad (\mathrm{kJ/kg}) \tag{2–8}$$

where $h = P/\rho + u$ is the enthalpy, V is the magnitude of velocity, and z is the elevation of the system relative to some external reference point.

By using the enthalpy instead of the internal energy to represent the energy of a flowing fluid, we do not need to be concerned about the flow work. The energy associated with pushing the fluid is automatically taken care of by enthalpy. In fact, this is the main reason for defining the property enthalpy.

The differential and finite changes in the internal energy and enthalpy of an *ideal gas* can be expressed in terms of the specific heats as

$$du = c_\upsilon\, dT \quad \text{and} \quad dh = c_p\, dT \tag{2–9}$$

where c_υ and c_p are the constant-volume and constant-pressure specific heats of the ideal gas. Using specific heat values at the average temperature, the finite changes in internal energy and enthalpy can be expressed approximately as

$$\Delta u \cong c_{\upsilon,\text{avg}}\, \Delta T \quad \text{and} \quad \Delta h \cong c_{p,\text{avg}}\, \Delta T \tag{2–10}$$

For *incompressible substances,* the constant-volume and constant-pressure specific heats are identical. Therefore, $c_p \cong c_\upsilon \cong c$ for liquids, and the change in the internal energy of liquids can be expressed as $\Delta u \cong c_{\text{avg}}\, \Delta T$.

Noting that $\rho =$ constant for incompressible substances, the differentiation of enthalpy $h = u + P/\rho$ gives $dh = du + dP/\rho$. Integrating, the enthalpy change becomes

$$\Delta h = \Delta u + \Delta P/\rho \cong c_{\text{avg}}\, \Delta T + \Delta P/\rho \tag{2–11}$$

Therefore, $\Delta h \cong \Delta u \cong c_{\text{avg}}\, \Delta T$ for constant-pressure processes, and $\Delta h = \Delta P/\rho$ for constant-temperature processes in liquids.

2–5 ■ COMPRESSIBILITY AND SPEED OF SOUND

Coefficient of Compressibility

We know from experience that the volume (or density) of a fluid changes with a change in its temperature or pressure. Fluids usually expand as they are heated or depressurized and contract as they are cooled or pressurized. But the amount of volume change is different for different fluids, and we need to define properties that relate volume changes to the changes in pressure and temperature. Two such properties are the bulk modulus of elasticity κ and the coefficient of volume expansion β.

It is a common observation that a fluid contracts when more pressure is applied on it and expands when the pressure acting on it is reduced (Fig. 2–11). That is, fluids act like elastic solids with respect to pressure. Therefore, in an analogous manner to Young's modulus of elasticity for solids, it is appropriate to define a **coefficient of compressibility** κ (also called the **bulk modulus of compressibility** or **bulk modulus of elasticity**) for fluids as

$$\kappa = -\mathsf{v}\left(\frac{\partial P}{\partial \mathsf{v}}\right)_T = \rho\left(\frac{\partial P}{\partial \rho}\right)_T \quad \text{(Pa)} \tag{2–12}$$

It can also be expressed approximately in terms of finite changes as

$$\kappa \cong -\frac{\Delta P}{\Delta \mathsf{v}/\mathsf{v}} \cong \frac{\Delta P}{\Delta \rho/\rho} \quad (T = \text{constant}) \tag{2–13}$$

Noting that $\Delta\mathsf{v}/\mathsf{v}$ or $\Delta\rho/\rho$ is dimensionless, κ must have the dimension of pressure (Pa or psi). Also, the coefficient of compressibility represents the change in pressure corresponding to a fractional change in volume or density of the fluid while the temperature remains constant. Then it follows that the coefficient of compressibility of a truly incompressible substance (v = constant) is infinity.

A large value of κ indicates that a large change in pressure is needed to cause a small fractional change in volume, and thus a fluid with a large κ is essentially incompressible. This is typical for liquids, and explains why liquids are usually considered to be *incompressible*. For example, the pressure of water at normal atmospheric conditions must be raised to 210 atm to compress it 1 percent, corresponding to a coefficient of compressibility value of $\kappa = 21{,}000$ atm.

Small density changes in liquids can still cause interesting phenomena in piping systems such as the *water hammer*—characterized by a sound that resembles the sound produced when a pipe is "hammered." This occurs when a liquid in a piping network encounters an abrupt flow restriction (such as a closing valve) and is locally compressed. The acoustic waves that are produced strike the pipe surfaces, bends, and valves as they propagate and reflect along the pipe, causing the pipe to vibrate and produce the familiar sound. In addition to the irritating sound, water hammering can be quite destructive, leading to leaks or even structural damage. The effect can be suppressed with a *water hammer arrestor*

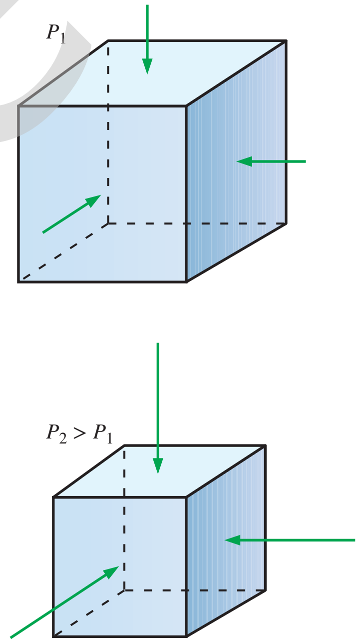


FIGURE 2–11
Fluids, like solids, compress when the applied pressure is increased from P_1 to P_2.

(*a*)

(*b*)

FIGURE 2–12
Water hammer arrestors:
(*a*) A large surge tower built to protect the pipeline against water hammer damage.
Photo by Arris S. Tijsseling, visitor of the University of Adelaide, Australia. Used by permission
(*b*) Much smaller arrestors used for supplying water to a household washing machine.
Photo provided courtesy of Oatey Company

(Fig. 2–12), which is a volumetric chamber containing either a bellows or piston to absorb the shock. For large pipes that carry liquids (e.g., hydropower penstocks), a vertical tube called a *surge tower* is often used. A surge tower has a free air surface at the top and is virtually maintenance free. For other pipelines, a closed *hydraulic accumulator tank* is used, in which a gas such as air or nitrogen is compressed in the accumulator tank to soften the shock.

Note that volume and pressure are inversely proportional (volume decreases as pressure is increased and thus $\partial P/\partial v$ is a negative quantity), and the negative sign in the definition (Eq. 2–12) ensures that κ is a positive quantity. Also, differentiating $\rho = 1/v$ gives $d\rho = -dv/v^2$, which can be rearranged as

$$\frac{d\rho}{\rho} = -\frac{dv}{v} \tag{2–14}$$

That is, the fractional changes in the specific volume and the density of a fluid are equal in magnitude but opposite in sign.

For an ideal gas, $P = \rho RT$ and $(\partial P/\partial \rho)_T = RT = P/\rho$, and thus

$$\kappa_{\text{ideal gas}} = P \qquad \text{(Pa)} \tag{2–15}$$

Therefore, the coefficient of compressibility of an ideal gas is equal to its absolute pressure, and the coefficient of compressibility of the gas increases with increasing pressure. Substituting $\kappa = P$ into the definition of the coefficient of compressibility and rearranging gives

Ideal gas: $$\frac{\Delta\rho}{\rho} = \frac{\Delta P}{P} \qquad (T = \text{constant}) \tag{2–16}$$

Therefore, the percent increase of density of an ideal gas during isothermal compression is equal to the percent increase in pressure.

For air at 1 atm pressure, $\kappa = P = 1$ atm and a decrease of 1 percent in volume ($\Delta V/V = -0.01$) corresponds to an increase of $\Delta P = 0.01$ atm in pressure. But for air at 1000 atm, $\kappa = 1000$ atm and a decrease of 1 percent in volume corresponds to an increase of $\Delta P = 10$ atm in pressure. Therefore, a small fractional change in the volume of a gas can cause a large change in pressure at very high pressures.

The inverse of the coefficient of compressibility is called the **isothermal compressibility** α and is expressed as

$$\alpha = \frac{1}{\kappa} = -\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_T = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial P}\right)_T \qquad \text{(1/Pa)} \tag{2–17}$$

The isothermal compressibility of a fluid represents the fractional change in volume or density corresponding to a unit change in pressure.

Coefficient of Volume Expansion

The density of a fluid, in general, depends more strongly on temperature than it does on pressure, and the variation of density with temperature is responsible for numerous natural phenomena such as winds, currents in oceans, rise of plumes in chimneys, the operation of hot-air balloons, heat transfer by natural convection, and even the rise of hot air and thus the phrase “heat rises”

(Fig. 2–13). To quantify these effects, we need a property that represents the *variation of the density of a fluid with temperature at constant pressure.*

The property that provides that information is the **coefficient of volume expansion** (or *volume expansivity*) β, defined as (Fig. 2–14)

$$\beta = \frac{1}{\mathrm{v}}\left(\frac{\partial \mathrm{v}}{\partial T}\right)_P = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \quad (1/\mathrm{K}) \tag{2–18}$$

It can also be expressed approximately in terms of finite changes as

$$\beta \approx \frac{\Delta \mathrm{v}/\mathrm{v}}{\Delta T} = -\frac{\Delta\rho/\rho}{\Delta T} \quad (\text{at constant } P) \tag{2–19}$$

A large value of β for a fluid means a large change in density with temperature, and the product $\beta\ \Delta T$ represents the fraction of volume change of a fluid that corresponds to a temperature change of ΔT at constant pressure.

It can be shown that the volume expansion coefficient of an *ideal gas* ($P = \rho RT$) at a temperature T is equivalent to the inverse of the temperature:

$$\beta_{\text{ideal gas}} = \frac{1}{T} \quad (1/\mathrm{K}) \tag{2–20}$$

where T is the *absolute* temperature.

In the study of natural convection currents, the condition of the main fluid body that surrounds the finite hot or cold regions is indicated by the subscript "infinity" to serve as a reminder that this is the value at a distance where the presence of the hot or cold region is not felt. In such cases, the volume expansion coefficient can be expressed approximately as

$$\beta \approx -\frac{(\rho_\infty - \rho)/\rho}{T_\infty - T} \quad \text{or} \quad \rho_\infty - \rho = \rho\beta(T - T_\infty) \tag{2–21}$$

where ρ_∞ is the density and T_∞ is the temperature of the quiescent fluid away from the confined hot or cold fluid pocket.

We will see in Chap. 3 that natural convection currents are initiated by the *buoyancy force,* which is proportional to the *density difference,* which is in turn proportional to the *temperature difference* at constant pressure. Therefore, the larger the temperature difference between the hot or cold fluid pocket and the surrounding main fluid body, the *larger* the buoyancy force and thus the *stronger* the natural convection currents. A related phenomenon sometimes occurs when an aircraft flies near the speed of sound. The sudden drop in temperature produces condensation of water vapor on a visible vapor cloud (Fig. 2–15).

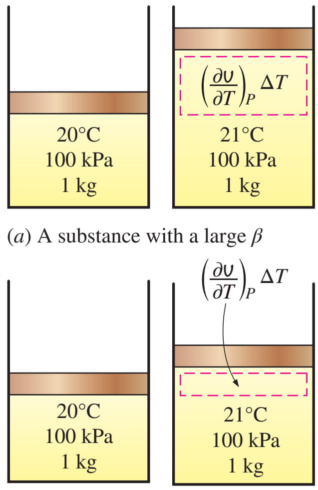
The combined effects of pressure and temperature changes on the volume change of a fluid can be determined by taking the specific volume to be a function of T and P. Differentiating $\mathrm{v} = \mathrm{v}(T, P)$ and using the definitions of the compression and expansion coefficients α and β give

$$d\mathrm{v} = \left(\frac{\partial \mathrm{v}}{\partial T}\right)_P dT + \left(\frac{\partial \mathrm{v}}{\partial P}\right)_T dP = (\beta\, dT - \alpha\, dP)\mathrm{v} \tag{2–22}$$

Then the fractional change in volume (or density) due to changes in pressure and temperature can be expressed approximately as

$$\frac{\Delta \mathrm{v}}{\mathrm{v}} = -\frac{\Delta\rho}{\rho} \cong \beta\,\Delta T - \alpha\,\Delta P \tag{2–23}$$

FIGURE 2–13
Natural convection over a woman's hand.



(*b*) A substance with a small β

FIGURE 2–14
The coefficient of volume expansion is a measure of the change in volume of a substance with temperature at constant pressure.

FIGURE 2–15
Vapor cloud around an F/A-18F Super Hornet as it flies near the speed of sound.
U.S. Navy photo by Photographer's Mate 3rd Class Jonathan Chandler.

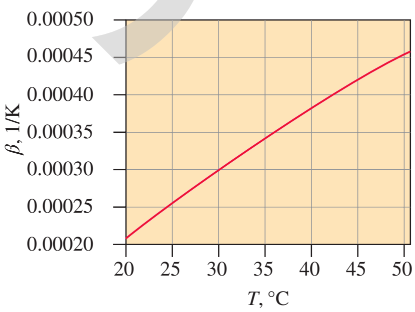


FIGURE 2–16
The variation of the coefficient of volume expansion β of water with temperature in the range of 20°C to 50°C.
Data were generated and plotted using EES.

EXAMPLE 2–3 Variation of Density with Temperature and Pressure

Consider water initially at 20°C and 1 atm. Determine the final density of the water (*a*) if it is heated to 50°C at a constant pressure of 1 atm, and (*b*) if it is compressed to 100-atm pressure at a constant temperature of 20°C. Take the isothermal compressibility of water to be $\alpha = 4.80 \times 10^{-5}$ atm^{-1}.

SOLUTION Water at a given temperature and pressure is considered. The densities of water after it is heated and after it is compressed are to be determined.

Assumptions **1** The coefficient of volume expansion and the isothermal compressibility of water are constant in the given temperature range. **2** An approximate analysis is performed by replacing differential changes in quantities by finite changes.

Properties The density of water at 20°C and 1 atm pressure is $\rho_1 = 998.0$ kg/m^3. The coefficient of volume expansion at the average temperature of $(20 + 50)/2 = 35$°C is $\beta = 0.337 \times 10^{-3}$ K^{-1}. The isothermal compressibility of water is given to be $\alpha = 4.80 \times 10^{-5}$ atm^{-1}.

Analysis When differential quantities are replaced by differences and the properties α and β are assumed to be constant, the change in density in terms of the changes in pressure and temperature is expressed approximately as (Eq. 2–23)

$$\Delta\rho = \alpha\rho\,\Delta P - \beta\rho\,\Delta T$$

(*a*) The change in density due to the change of temperature from 20°C to 50°C at constant pressure is

$$\Delta\rho = -\beta\rho\,\Delta T = -(0.337 \times 10^{-3}\ \text{K}^{-1})(998\ \text{kg/m}^3)(50 - 20)\ \text{K}$$
$$= -10.0\ \text{kg/m}^3$$

Noting that $\Delta\rho = \rho_2 - \rho_1$, the density of water at 50°C and 1 atm is

$$\rho_2 = \rho_1 + \Delta\rho = 998.0 + (-10.0) = \mathbf{988.0\ kg/m^3}$$

which is almost identical to the listed value of 988.1 kg/m^3 at 50°C in Table A–3. This is mostly due to β varying with temperature almost linearly, as shown in Fig. 2–16.

(*b*) The change in density due to a change of pressure from 1 atm to 100 atm at constant temperature is

$$\Delta\rho = \alpha\rho\,\Delta P = (4.80 \times 10^{-5}\ \text{atm}^{-1})(998\ \text{kg/m}^3)(100 - 1)\ \text{atm} = 4.7\ \text{kg/m}^3$$

Then the density of water at 100 atm and 20°C becomes

$$\rho_2 = \rho_1 + \Delta\rho = 998.0 + 4.7 = \mathbf{1002.7\ kg/m^3}$$

Discussion Note that the density of water decreases while being heated and increases while being compressed, as expected. This problem can be solved more accurately using differential analysis when functional forms of properties are available.

Speed of Sound and Mach Number

An important parameter in the study of compressible flow is the **speed of sound** (or the **sonic speed**), defined as the speed at which an infinitesimally small pressure wave travels through a medium. The pressure wave may be caused by a small disturbance, which creates a slight change in local pressure.

To obtain a relation for the speed of sound in a medium, consider a duct that is filled with a fluid at rest, as shown in Fig. 2–17. A piston fitted in the duct is now moved to the right with a constant incremental velocity dV, creating a sonic wave. The wave front moves to the right through the fluid at the speed of sound c and separates the moving fluid adjacent to the piston from the fluid still at rest. The fluid to the left of the wave front experiences an incremental change in its thermodynamic properties, while the fluid on the right of the wave front maintains its original thermodynamic properties, as shown in Fig. 2–17.

To simplify the analysis, consider a control volume that encloses the wave front and moves with it, as shown in Fig. 2–18. To an observer traveling with the wave front, the fluid to the right appears to be moving toward the wave front with a speed of c and the fluid to the left to be moving away from the wave front with a speed of $c - dV$. Of course, the observer sees the control volume that encloses the wave front (and herself or himself) as stationary, and the observer is witnessing a steady-flow process. The mass balance for this single-stream, steady-flow process is expressed as

$$\dot{m}_{\text{right}} = \dot{m}_{\text{left}}$$

or

$$\rho A c = (\rho + d\rho)A(c - dV)$$

By canceling the cross-sectional (or flow) area A and neglecting the higher-order terms, this equation reduces to

$$c\,d\rho - \rho\,dV = 0$$

No heat or work crosses the boundaries of the control volume during this steady-flow process, and the potential energy change can be neglected. Then the steady-flow energy balance $e_{\text{in}} = e_{\text{out}}$ becomes

$$h + \frac{c^2}{2} = h + dh + \frac{(c - dV)^2}{2}$$

which yields

$$dh - c\,dV = 0$$

where we have neglected the second-order term $(dV)^2$. The amplitude of the ordinary sonic wave is very small and does not cause any appreciable change in the pressure and temperature of the fluid. Therefore, the propagation of a sonic wave is not only adiabatic but also very nearly isentropic. Then the thermodynamic relation $T\,ds = dh - dP/\rho$ (see Çengel and Boles, 2015) reduces to

$$\cancel{T\,ds}^{\,0} = dh - \frac{dP}{\rho}$$

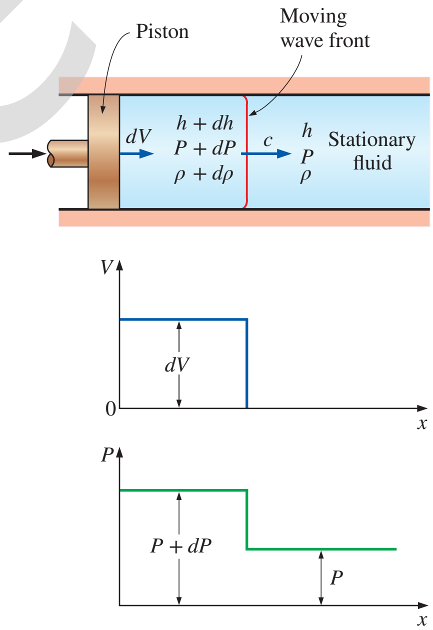


FIGURE 2–17
Propagation of a small pressure wave along a duct.

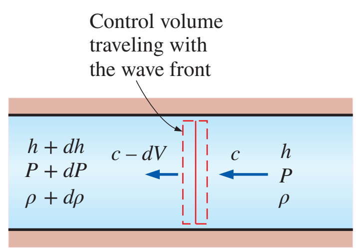


FIGURE 2–18
Control volume moving with the small pressure wave along a duct.

FIGURE 2–19
The speed of sound in air increases with temperature. At typical outside temperatures, c is about 340 m/s. In round numbers, therefore, the sound of thunder from a lightning strike travels about 1 km in 3 seconds. If you see the lightning and then hear the thunder less than 3 seconds later, you know that the lightning is close, and it is time to go indoors!

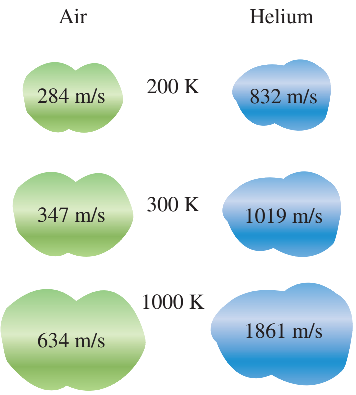


FIGURE 2–20
The speed of sound changes with temperature and varies with the fluid.

or

$$dh = \frac{dP}{\rho}$$

Combining the above equations yields the desired expression for the speed of sound as

$$c^2 = \frac{dP}{d\rho} \qquad \text{at } s = \text{constant}$$

or

$$c^2 = \left(\frac{\partial P}{\partial \rho}\right)_s \tag{2–24}$$

It is left as an exercise for the reader to show, by using thermodynamic property relations, that Eq. 2–24 can also be written as

$$c^2 = k\left(\frac{\partial P}{\partial \rho}\right)_T \tag{2–25}$$

where $k = c_p/c_v$ is the specific heat ratio of the fluid. Note that the speed of sound in a fluid is a function of the thermodynamic properties of that fluid Fig. 2–19.

When the fluid is an ideal gas ($P = \rho RT$), the differentiation in Eq. 2–25 can be performed to yield

$$c^2 = k\left(\frac{\partial P}{\partial \rho}\right)_T = k\left[\frac{\partial(\rho RT)}{\partial \rho}\right]_T = kRT$$

or

$$c = \sqrt{kRT} \tag{2–26}$$

Noting that the gas constant R has a fixed value for a specified ideal gas and the specific heat ratio k of an ideal gas is, at most, a function of temperature, we see that the speed of sound in a specified ideal gas is a function of temperature alone (Fig. 2–20).

A second important parameter in the analysis of compressible fluid flow is the **Mach number** Ma, named after the Austrian physicist Ernst Mach (1838–1916). It is the ratio of the actual speed of the fluid (or an object in still fluid) to the speed of sound in the same fluid at the same state:

$$\text{Ma} = \frac{V}{c} \tag{2–27}$$

Mach number can also be defined as the ratio of inertial forces to elastic forces. If Ma is less than about 1/3, the flow may be approximated as incompressible since the effects of compressibility become significant only when the Mach number exceeds this value.

Note that the Mach number depends on the speed of sound, which depends on the state of the fluid. Therefore, the Mach number of an aircraft cruising at constant velocity in still air may be different at different locations (Fig. 2–21).

Fluid flow regimes are often described in terms of the flow Mach number. The flow is called **sonic** when Ma = 1, **subsonic** when Ma < 1, **supersonic** when Ma > 1, **hypersonic** when Ma ≫ 1, and **transonic** when Ma ≅ 1.

EXAMPLE 2–4 Mach Number of Air Entering a Diffuser

Air enters a diffuser shown in Fig. 2–22 with a speed of 200 m/s. Determine (*a*) the speed of sound and (*b*) the Mach number at the diffuser inlet when the air temperature is 30°C.

SOLUTION Air enters a diffuser at high speed. The speed of sound and the Mach number are to be determined at the diffuser inlet.

Assumption Air at the specified conditions behaves as an ideal gas.

Properties The gas constant of air is $R = 0.287$ kJ/kg·K, and its specific heat ratio at 30°C is 1.4.

Analysis We note that the speed of sound in a gas varies with temperature, which is given to be 30°C.

(*a*) The speed of sound in air at 30°C is determined from Eq. 2–26 to be

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.287\text{ kJ/kg·K})(303\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = \mathbf{349\ m/s}$$

(*b*) Then the Mach number becomes

$$\text{Ma} = \frac{V}{c} = \frac{200\text{ m/s}}{349\text{ m/s}} = \mathbf{0.573}$$

Discussion The flow at the diffuser inlet is subsonic since Ma < 1.

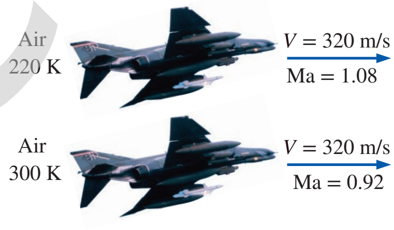


FIGURE 2–21
The Mach number can be different at different temperatures even if the flight speed is the same.
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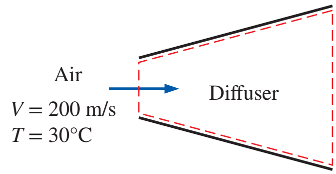


FIGURE 2–22
Schematic for Example 12–4.

2–6 ▪ VISCOSITY

When two solid bodies in contact move relative to each other, a friction force develops at the contact surface in the direction opposite to motion. To move a table on the floor, for example, we have to apply a force to the table in the horizontal direction large enough to overcome the friction force. The magnitude of the force needed to move the table depends on the *friction coefficient* between the table legs and the floor.

The situation is similar when a fluid moves relative to a solid or when two fluids move relative to each other. We move with relative ease in air, but not so in water. Moving in oil would be even more difficult, as can be observed by the slower downward motion of a glass ball dropped in a tube filled with oil. It appears that there is a property that represents the internal resistance of a fluid to motion or the "fluidity," and that property is the **viscosity**. The force a flowing fluid exerts on a body in the flow direction is called the **drag force**, and the magnitude of this force depends, in part, on viscosity (Fig. 2–23).

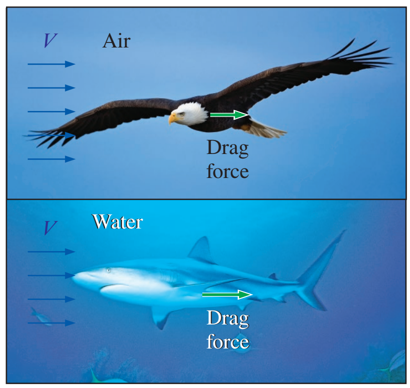


FIGURE 2–23
A fluid moving relative to a body exerts a drag force on the body, partly because of friction caused by viscosity.
Top: © Photodisc/Getty Images RF
Bottom: © Digital Vision/Getty Images RF

To obtain a relation for viscosity, consider a fluid layer between two very large parallel plates (or equivalently, two parallel plates immersed in a large body of a fluid) separated by a distance ℓ (Fig. 2–24). Now a constant parallel force F is applied to the upper plate while the lower plate is held fixed. After the initial transients, it is observed that the upper plate moves continuously under the influence of this force at a constant speed V. The fluid in contact with the upper plate sticks to the plate surface and moves with it at the same speed, and the shear stress τ acting on this fluid layer is

$$\tau = \frac{F}{A} \qquad \textbf{(2–28)}$$

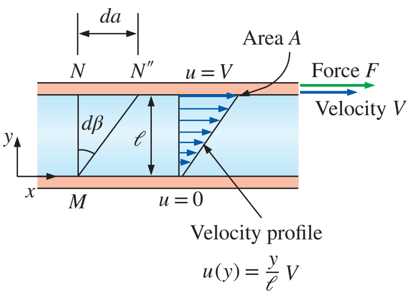


FIGURE 2–24
The behavior of a fluid in laminar flow between two parallel plates when the upper plate moves with a constant velocity.

where A is the contact area between the plate and the fluid. Note that the fluid layer deforms continuously under the influence of shear stress.

The fluid in contact with the lower plate assumes the velocity of that plate, which is zero (because of the no-slip condition—see Section 1–2). In steady laminar flow, the fluid velocity between the plates varies linearly between 0 and V, and thus the *velocity profile* and the *velocity gradient* are

$$u(y) = \frac{y}{\ell}V \quad \text{and} \quad \frac{du}{dy} = \frac{V}{\ell} \tag{2–29}$$

where y is the vertical distance from the lower plate.

During a differential time interval dt, the sides of fluid particles along a vertical line MN rotate through a differential angle $d\beta$ while the upper plate moves a differential distance $da = V\,dt$. The angular displacement or deformation (or shear strain) can be expressed as

$$d\beta \approx \tan d\beta = \frac{da}{\ell} = \frac{V\,dt}{\ell} = \frac{du}{dy}dt \tag{2–30}$$

Rearranging, the rate of deformation under the influence of shear stress τ becomes

$$\frac{d\beta}{dt} = \frac{du}{dy} \tag{2–31}$$

Thus we conclude that the rate of deformation of a fluid element is equivalent to the velocity gradient du/dy. Further, it can be verified experimentally that for most fluids the rate of deformation (and thus the velocity gradient) is directly proportional to the shear stress τ,

$$\tau \propto \frac{d\beta}{dt} \quad \text{or} \quad \tau \propto \frac{du}{dy} \tag{2–32}$$

Fluids for which the rate of deformation is linearly proportional to the shear stress are called **Newtonian fluids** after Sir Isaac Newton, who expressed it first in 1687. Most common fluids such as water, air, gasoline, and oils are Newtonian fluids. Blood and liquid plastics are examples of non-Newtonian fluids.

In one-dimensional shear flow of Newtonian fluids, shear stress can be expressed by the linear relationship

Shear stress:
$$\tau = \mu\frac{du}{dy} \quad (\text{N/m}^2) \tag{2–33}$$

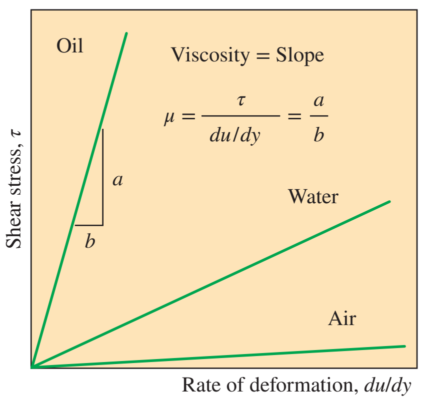


FIGURE 2–25
The rate of deformation (velocity gradient) of a Newtonian fluid is proportional to shear stress, and the constant of proportionality is the viscosity.

where the constant of proportionality μ is called the **coefficient of viscosity** or the **dynamic** (or **absolute**) **viscosity** of the fluid, whose unit is kg/m·s, or equivalently, N·s/m^2 (or Pa·s where Pa is the pressure unit pascal). A common viscosity unit is **poise**, which is equivalent to 0.1 Pa·s (or *centipoise,* which is one-hundredth of a poise). The viscosity of water at 20°C is 1.002 centipoise, and thus the unit centipoise serves as a useful reference. A plot of shear stress versus the rate of deformation (velocity gradient) for a Newtonian fluid is a straight line whose slope is the viscosity of the fluid, as shown in Fig. 2–25. Note that viscosity is independent of the rate of deformation for Newtonian fluids. Since the rate of deformation is proportional to the strain rate, Fig. 2–25 reveals that viscosity is actually a coefficient in a stress–strain relationship.

The **shear force** acting on a Newtonian fluid layer (or, by Newton's third law, the force acting on the plate) is

Shear force: $$F = \tau A = \mu A \frac{du}{dy} \quad \text{(N)} \tag{2–34}$$

where again A is the contact area between the plate and the fluid. Then the force F required to move the upper plate in Fig. 2–24 at a constant speed of V while the lower plate remains stationary is

$$F = \mu A \frac{V}{\ell} \quad \text{(N)} \tag{2–35}$$

This relation can alternately be used to calculate μ when the force F is measured. Therefore, the experimental setup just described can be used to measure the viscosity of fluids. Note that under identical conditions, the force F would be very different for different fluids.

For non-Newtonian fluids, the relationship between shear stress and rate of deformation is not linear, as shown in Fig. 2–26. The slope of the curve on the τ versus du/dy chart is referred to as the *apparent viscosity* of the fluid. Fluids for which the apparent viscosity increases with the rate of deformation (such as solutions with suspended starch or sand) are referred to as *dilatant* or *shear thickening fluids,* and those that exhibit the opposite behavior (the fluid becoming less viscous as it is sheared harder, such as some paints, polymer solutions, and fluids with suspended particles) are referred to as *pseudoplastic* or *shear thinning fluids*. Some materials such as toothpaste can resist a finite shear stress and thus behave as a solid, but deform continuously when the shear stress exceeds the yield stress and behave as a fluid. Such materials are referred to as Bingham plastics after Eugene C. Bingham (1878–1945), who did pioneering work on fluid viscosity for the U.S. National Bureau of Standards in the early twentieth century.

In fluid mechanics and heat transfer, the ratio of dynamic viscosity to density appears frequently. For convenience, this ratio is given the name **kinematic viscosity** ν and is expressed as $\nu = \mu/\rho$. Two common units of kinematic viscosity are m^2/s and **stoke** (1 stoke = 1 cm^2/s = 0.0001 m^2/s).

In general, the viscosity of a fluid depends on both temperature and pressure, although the dependence on pressure is rather weak. For *liquids*, both the dynamic and kinematic viscosities are practically independent of pressure, and any small variation with pressure is usually disregarded, except at extremely high pressures. For *gases,* this is also the case for dynamic viscosity (at low to moderate pressures), but not for kinematic viscosity since the density of a gas is proportional to its pressure (Fig. 2–27).

The viscosity of a fluid is a measure of its resistance to the rate of deformation. Viscosity is due to the internal frictional force that develops between different layers of fluids as they are forced to move relative to each other.

The viscosity of a fluid is directly related to the pumping power needed to transport a fluid in a pipe or to move a body (such as a car in air or a submarine in the sea) through a fluid. Viscosity is caused by the cohesive forces between the molecules in liquids and by the molecular collisions in gases, and it varies greatly with temperature. The viscosity of liquids decreases with temperature, whereas the viscosity of gases increases with

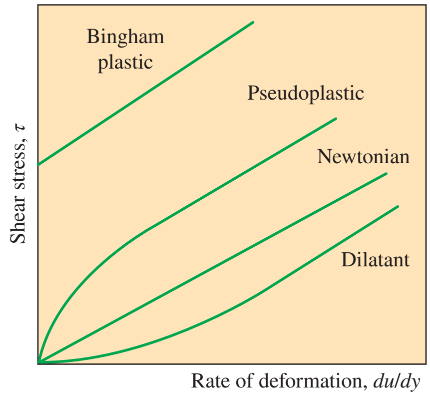


FIGURE 2–26
Variation of shear stress with the rate of deformation for Newtonian and non-Newtonian fluids (the slope of a curve at a point is the apparent viscosity of the fluid at that point).

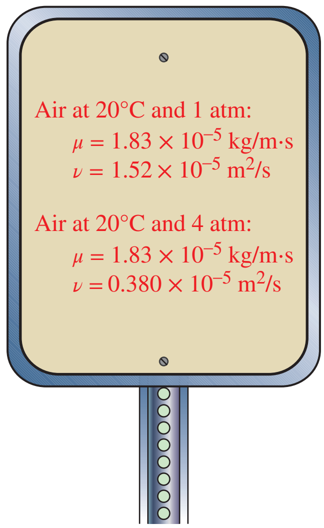


FIGURE 2–27
Dynamic viscosity, in general, does not depend on pressure, but kinematic viscosity does.

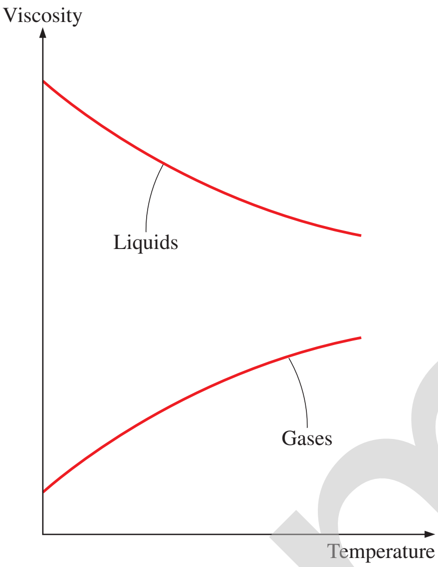


FIGURE 2–28
The viscosity of liquids decreases and the viscosity of gases increases with temperature.

TABLE 2–3
Dynamic viscosity of some fluids at 1 atm and 20°C (unless otherwise stated)

Fluid	Dynamic Viscosity μ, kg/m·s
Glycerin:	
−20°C	134.0
0°C	10.5
20°C	1.52
40°C	0.31
Engine oil:	
SAE 10W	0.10
SAE 10W30	0.17
SAE 30	0.29
SAE 50	0.86
Mercury	0.0015
Ethyl alcohol	0.0012
Water:	
0°C	0.0018
20°C	0.0010
100°C (liquid)	0.00028
100°C (vapor)	0.000012
Blood, 37°C	0.00040
Gasoline	0.00029
Ammonia	0.00015
Air	0.000018
Hydrogen, 0°C	0.0000088

temperature (Fig. 2–28). This is because in a liquid the molecules possess more energy at higher temperatures, and they can oppose the large cohesive intermolecular forces more strongly. As a result, the energized liquid molecules can move more freely.

In a gas, on the other hand, the intermolecular forces are negligible, and the gas molecules at high temperatures move randomly at higher velocities. This results in more molecular collisions per unit volume per unit time and therefore in greater resistance to flow. The kinetic theory of gases predicts the viscosity of gases to be proportional to the square root of temperature. That is, $\mu_{gas} \propto \sqrt{T}$. This prediction is confirmed by practical observations, but deviations for different gases need to be accounted for by incorporating some correction factors. The viscosity of gases is expressed as a function of temperature by the Sutherland correlation (from The U.S. Standard Atmosphere) as

Gases:
$$\mu = \frac{aT^{1/2}}{1 + b/T} \quad \textbf{(2–36)}$$

where T is absolute temperature and a and b are experimentally determined constants. Note that measuring viscosity at two different temperatures is sufficient to determine these constants. For air at atmospheric conditions, the values of these constants are $a = 1.458 \times 10^{-6}$ kg/(m·s·K$^{1/2}$) and $b = 110.4$ K. The viscosity of gases is independent of pressure at low to moderate pressures (from a few percent of 1 atm to several atm). But viscosity increases at high pressures due to the increase in density.

For liquids, the viscosity is approximated as

Liquids:
$$\mu = a10^{b/(T-c)} \quad \textbf{(2–37)}$$

where again T is absolute temperature and a, b, and c are experimentally determined constants. For water, using the values $a = 2.414 \times 10^{-5}$ N·s/m^2, $b = 247.8$ K, and $c = 140$ K results in less than 2.5 percent error in viscosity in the temperature range of 0°C to 370°C (Touloukian et al., 1975).

The viscosities of some fluids at room temperature are listed in Table 2–3. They are plotted against temperature in Fig. 2–29. Note that the viscosities of different fluids differ by several orders of magnitude. Also note that it is more difficult to move an object in a higher-viscosity fluid such as engine oil than it is in a lower-viscosity fluid such as water. Liquids, in general, are much more viscous than gases.

Consider a fluid layer of thickness ℓ within a small gap between two concentric cylinders, such as the thin layer of oil in a journal bearing. The gap between the cylinders can be modeled as two parallel flat plates separated by the fluid. Noting that torque is $\mathrm{T} = FR$ (force times the moment arm, which is the radius R of the inner cylinder in this case), the tangential velocity is $V = \omega R$ (angular velocity times the radius), and taking the wetted surface area of the inner cylinder to be $A = 2\pi RL$ by disregarding the shear stress acting on the two ends of the inner cylinder, torque can be expressed as

$$\mathrm{T} = FR = \mu \frac{2\pi R^3 \omega L}{\ell} = \mu \frac{4\pi^2 R^3 \dot{n} L}{\ell} \quad \textbf{(2–38)}$$

where L is the length of the cylinder and $\dot{n}$ is the number of revolutions per unit time, which is usually expressed in rpm (revolutions per minute). Note

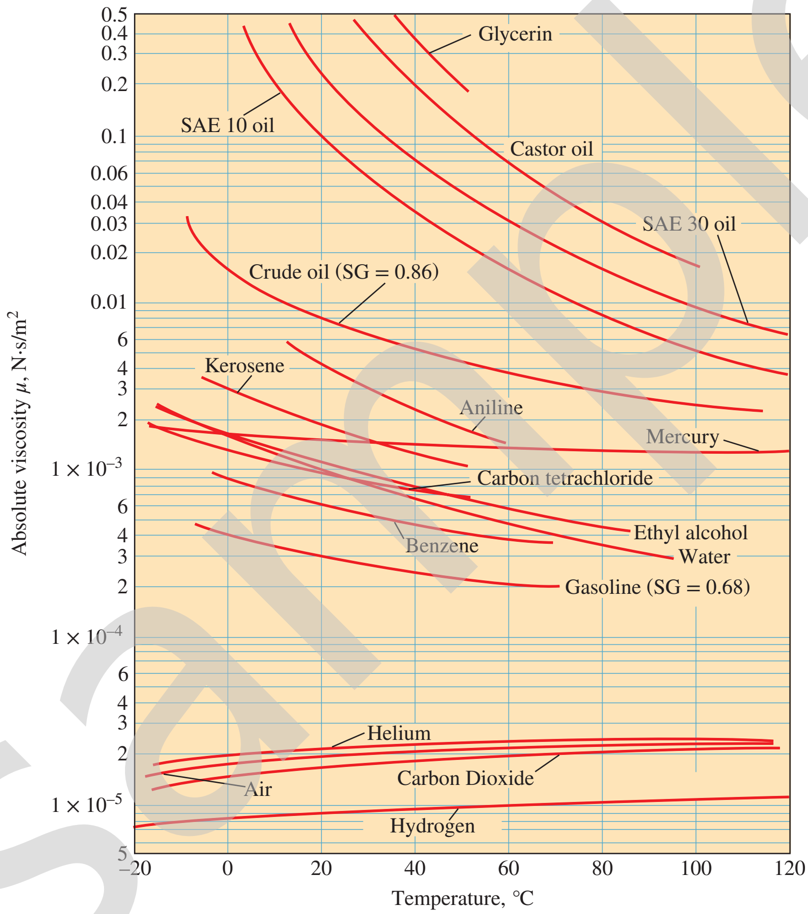


FIGURE 2–29
The variation of dynamic (absolute) viscosity of common fluids with temperature at 1 atm (1 N·s/m^2 = 1 kg/m·s = 0.020886 lbf·s/ft^2).

Data from EES and F. M. White, Fluid Mechanics *7e. Copyright © 2011 The McGraw-Hill Companies, Inc.*

that the angular distance traveled during one rotation is 2π rad, and thus the relation between the angular velocity in rad/min and the rpm is $\omega = 2\pi\dot{n}$. Equation 2–38 can be used to calculate the viscosity of a fluid by measuring torque at a specified angular velocity. Therefore, two concentric cylinders can be used as a *viscometer,* a device that measures viscosity.

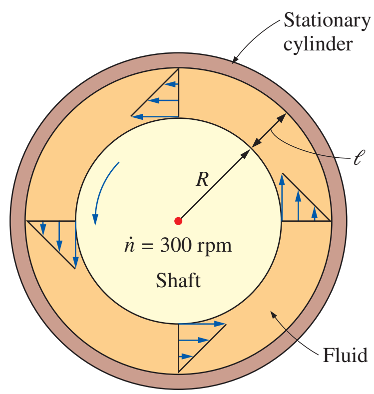


FIGURE 2–30
Schematic for Example 2–5 (not to scale).

EXAMPLE 2–5 Determining the Viscosity of a Fluid

The viscosity of a fluid is to be measured by a viscometer constructed of two 40-cm-long concentric cylinders (Fig. 2–30). The outer diameter of the inner cylinder is 12 cm, and the gap between the two cylinders is 0.15 cm. The inner cylinder is rotated at 300 rpm, and the torque is measured to be 1.8 N·m. Determine the viscosity of the fluid.

SOLUTION The torque and the rpm of a double cylinder viscometer are given. The viscosity of the fluid is to be determined.

Assumptions **1** The inner cylinder is completely submerged in the fluid. **2** The viscous effects on the two ends of the inner cylinder are negligible.

Analysis The velocity profile is linear only when the curvature effects are negligible, and the profile can be approximated as being linear in this case since $\ell/R = 0.025 \ll 1$. Solving Eq. 2–38 for viscosity and substituting the given values, the viscosity of the fluid is determined to be

$$\mu = \frac{T\ell}{4\pi^2 R^3 \dot{n} L} = \frac{(1.8\ \text{N}\cdot\text{m})(0.0015\ \text{m})}{4\pi^2(0.06\ \text{m})^3\left(300\ \dfrac{1}{\text{min}}\right)\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right)(0.4\ \text{m})} = \mathbf{0.158\ N\cdot s/m^2}$$

Discussion Viscosity is a strong function of temperature, and a viscosity value without a corresponding temperature is of little usefulness. Therefore, the temperature of the fluid should have also been measured during this experiment, and reported with this calculation.

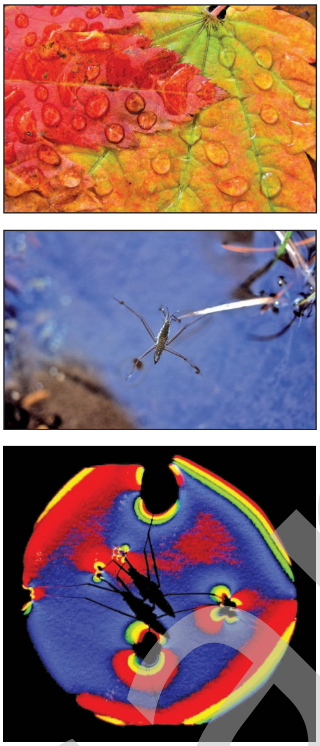


FIGURE 2–31
Some consequences of surface tension: (*a*) drops of water beading up on a leaf, (*b*) a water strider sitting on top of the surface of water, and (*c*) a color schlieren image of the water strider revealing how the water surface dips down where its feet contact the water (it looks like two insects but the second one is just a shadow).

(a) © Don Paulson Photography/Purestock/SuperStock RF
(b) NPS Photo by Rosalie LaRue
(c) © G.S. Settles, Gas Dynamics Lab, Penn State University. Used with permission.

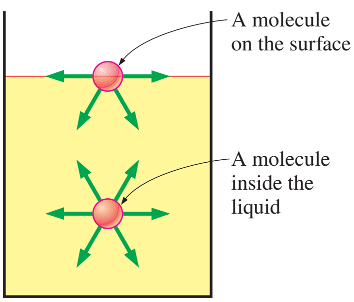


FIGURE 2–32
Attractive forces acting on a liquid molecule at the surface and deep inside the liquid.

2–7 ▪ SURFACE TENSION AND CAPILLARY EFFECT

It is often observed that a drop of blood forms a hump on a horizontal glass; a drop of mercury forms a near-perfect sphere and can be rolled just like a steel ball over a smooth surface; water droplets from rain or dew hang from branches or leaves of trees; a liquid fuel injected into an engine forms a mist of spherical droplets; water dripping from a leaky faucet falls as nearly spherical droplets; a soap bubble released into the air forms a nearly spherical shape; and water beads up into small drops on flower petals (Fig. 2–31*a*).

In these and other observances, liquid droplets behave like small balloons filled with the liquid, and the surface of the liquid acts like a stretched elastic membrane under tension. The pulling force that causes this tension acts parallel to the surface and is due to the attractive forces between the molecules of the liquid. The magnitude of this force per unit length is called **surface tension** or *coefficient of surface tension* σ_s and is usually expressed in the unit N/m (or lbf/ft in English units). This effect is also called *surface energy* (per unit area) and is expressed in the equivalent unit of $\text{N}\cdot\text{m/m}^2$ or J/m^2. In this case, σ_s represents the stretching work that needs to be done to increase the surface area of the liquid by a unit amount.

To visualize how surface tension arises, we present a microscopic view in Fig. 2–32 by considering two liquid molecules, one at the surface and one deep within the liquid body. The attractive forces applied on the interior molecule by the surrounding molecules balance each other because of symmetry. But the attractive forces acting on the surface molecule are not symmetric, and the attractive forces applied by the gas molecules above are usually very small. Therefore, there is a net attractive force acting on the molecule at the surface of the liquid, which tends to pull the molecules on the surface toward the interior of the liquid. This force is balanced by the repulsive forces from the molecules below the surface that are trying to be compressed. The result is that the liquid minimizes its surface area. This is the reason for the tendency of liquid droplets to attain a spherical shape, which has the minimum surface area for a given volume.

You also may have observed, with amusement, that some insects can land on water or even walk on water (Fig. 2–31*b*) and that small steel needles can float on water. These phenomena are made possible by surface tension which balances the weights of these objects.

To understand the surface tension effect better, consider a liquid film (such as the film of a soap bubble) suspended on a U-shaped wire frame with a movable side (Fig. 2–33). Normally, the liquid film tends to pull the movable wire inward in order to minimize its surface area. A force F needs to be applied on the movable wire in the opposite direction to balance this pulling effect. Both sides of the thin film are surfaces exposed to air, and thus the length along which the surface tension acts in this case is $2b$. Then a force balance on the movable wire gives $F = 2b\sigma_s$, and thus the surface tension can be expressed as

$$\sigma_s = \frac{F}{2b} \tag{2–39}$$

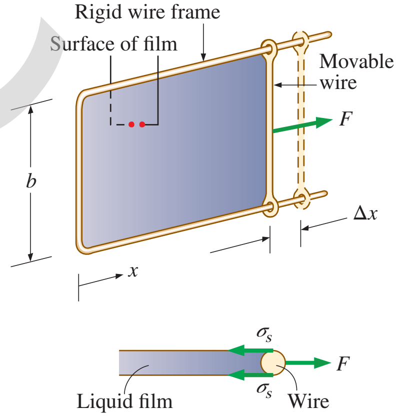


FIGURE 2–33
Stretching a liquid film with a U-shaped wire, and the forces acting on the movable wire of length *b*.

Note that for $b = 0.5$ m, the measured force F (in N) is simply the surface tension in N/m. An apparatus of this kind with sufficient precision can be used to measure the surface tension of various liquids.

In the U-shaped wire frame apparatus, the movable wire is pulled to stretch the film and increase its surface area. When the movable wire is pulled a distance Δx, the surface area increases by $\Delta A = 2b\ \Delta x$, and the work W done during this stretching process is

$$W = \text{Force} \times \text{Distance} = F\,\Delta x = 2b\sigma_s\,\Delta x = \sigma_s\,\Delta A$$

where we have assumed that the force remains constant over the small distance. This result can also be interpreted as *the surface energy of the film is increased by an amount $\sigma_s\ \Delta A$ during this stretching process,* which is consistent with the alternative interpretation of σ_s as surface energy per unit area. This is similar to a rubber band having more potential (elastic) energy after it is stretched further. In the case of liquid film, the work is used to move liquid molecules from the interior parts to the surface against the attraction forces of other molecules. Therefore, surface tension also can be defined as *the work done per unit increase in the surface area of the liquid*.

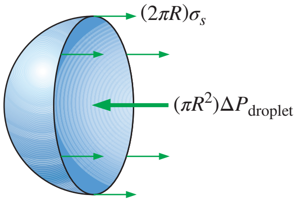
The surface tension varies greatly from substance to substance, and with temperature for a given substance, as shown in Table 2–4. At 20°C, for example, the surface tension is 0.073 N/m for water and 0.440 N/m for mercury surrounded by atmospheric air. The surface tension of mercury is large enough that mercury droplets form nearly spherical balls that can be rolled like a solid ball on a smooth surface. The surface tension of a liquid, in general, decreases with temperature and becomes zero at the critical point (and thus there is no distinct liquid–vapor interface at temperatures above the critical point). The effect of pressure on surface tension is usually negligible.

The surface tension of a substance can be changed considerably by *impurities*. Therefore, certain chemicals, called *surfactants,* can be added to a liquid to decrease its surface tension. For example, soaps and detergents lower the surface tension of water and enable it to penetrate the small openings between fibers for more effective washing. But this also

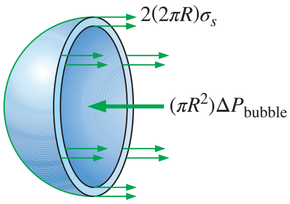
TABLE 2–4
Surface tension of some fluids in air at 1 atm and 20°C (unless otherwise stated)

Fluid	Surface Tension σ_s, N/m*
†Water:	
0°C	0.076
20°C	0.073
100°C	0.059
300°C	0.014
Glycerin	0.063
SAE 30 oil	0.035
Mercury	0.440
Ethyl alcohol	0.023
Blood, 37°C	0.058
Gasoline	0.022
Ammonia	0.021
Soap solution	0.025
Kerosene	0.028

* Multiply by 0.06852 to convert to lbf/ft.
† See Appendices for more precise data for water.



(*a*) Half of a droplet or air bubble



(*b*) Half of a soap bubble

FIGURE 2–34
The free-body diagram of half of a droplet or air bubble and half of a soap bubble.

means that devices whose operation depends on surface tension (such as heat pipes) can be destroyed by the presence of impurities due to poor workmanship.

We speak of surface tension for liquids only at liquid–liquid or liquid–gas interfaces. Therefore, it is imperative that the adjacent liquid or gas be specified when specifying surface tension. Surface tension determines the size of the liquid droplets that form, and so a droplet that keeps growing by the addition of more mass breaks down when the surface tension can no longer hold it together. This is like a balloon that bursts while being inflated when the pressure inside rises above the strength of the balloon material.

A curved interface indicates a pressure difference (or "pressure jump") across the interface with pressure being higher on the concave side. Consider, for example, a droplet of liquid in air, an air (or other gas) bubble in water, or a soap bubble in air. The excess pressure ΔP above atmospheric pressure can be determined by considering a free-body diagram of half the droplet or bubble (Fig. 2–34). Noting that surface tension acts along the circumference and the pressure acts on the area, horizontal force balances for the droplet or air bubble and the soap bubble give

$$\textit{Droplet or air bubble:}\quad (2\pi R)\sigma_s = (\pi R^2)\Delta P_{\text{droplet}} \rightarrow \Delta P_{\text{droplet}} = P_i - P_o = \frac{2\sigma_s}{R} \tag{2–40}$$

$$\textit{Soap bubble:}\quad 2(2\pi R)\sigma_s = (\pi R^2)\Delta P_{\text{bubble}} \rightarrow \Delta P_{\text{bubble}} = P_i - P_o = \frac{4\sigma_s}{R} \tag{2–41}$$

where P_i and P_o are the pressures inside and outside the droplet or bubble, respectively. When the droplet or bubble is in the atmosphere, P_o is simply atmospheric pressure. The extra factor of 2 in the force balance for the soap bubble is due to the existence of a soap film with *two* surfaces (inner and outer surfaces) and thus two circumferences in the cross section.

The excess pressure in a droplet of liquid in a gas (or a bubble of gas in a liquid) can also be determined by considering a differential increase in the radius of the droplet due to the addition of a differential amount of mass and interpreting the surface tension as the increase in the surface energy per unit area. Then the increase in the surface energy of the droplet during this differential expansion process becomes

$$\delta W_{\text{surface}} = \sigma_s\, dA = \sigma_s\, d(4\pi R^2) = 8\pi R\sigma_s\, dR$$

The expansion work done during this differential process is determined by multiplying the force by distance to obtain

$$\delta W_{\text{expansion}} = \text{Force} \times \text{Distance} = F\, dR = (\Delta PA)\, dR = 4\pi R^2\, \Delta P\, dR$$

Equating the two expressions above gives $\Delta P_{\text{droplet}} = 2\sigma_s/R$, which is the same relation obtained before and given in Eq. 2–40. Note that the excess pressure in a droplet or bubble is inversely proportional to the radius.

Capillary Effect

Another interesting consequence of surface tension is the **capillary effect**, which is the rise or fall of a liquid in a small-diameter tube inserted into the liquid. Such narrow tubes or confined flow channels are called **capillaries**. The rise of kerosene through a cotton wick inserted into the reservoir of a kerosene lamp is due to this effect. The capillary effect is also partially responsible for the rise of water to the top of tall trees. The curved free surface of a liquid in a capillary tube is called the **meniscus**.

It is commonly observed that water in a glass container curves up slightly at the edges where it touches the glass surface; but the opposite occurs for mercury: it curves down at the edges (Fig. 2–35). This effect is usually expressed by saying that water *wets* the glass (by sticking to it) while mercury does not. The strength of the capillary effect is quantified by the **contact** (or *wetting*) **angle** ϕ, defined as *the angle that the tangent to the liquid surface makes with the solid surface at the point of contact*. The surface tension force acts along this tangent line toward the solid surface. A liquid is said to wet the surface when $\phi < 90°$ and not to wet the surface when $\phi > 90°$. In atmospheric air, the contact angle of water (and most other organic liquids) with glass is nearly zero, $\phi \approx 0°$ (Fig. 2–36). Therefore, the surface tension force acts upward on water in a glass tube along the circumference, tending to pull the water up. As a result, water rises in the tube until the weight of the liquid in the tube above the liquid level of the reservoir balances the surface tension force. The contact angle is 130° for mercury–glass and 26° for kerosene–glass in air. Note that the contact angle, in general, is different in different environments (such as another gas or liquid in place of air).

The phenomenon of the capillary effect can be explained microscopically by considering *cohesive forces* (the forces between like molecules, such as water and water) and *adhesive forces* (the forces between unlike molecules, such as water and glass). The liquid molecules at the solid–liquid interface are subjected to both cohesive forces by other liquid molecules and adhesive forces by the molecules of the solid. The relative magnitudes of these forces determine whether a liquid wets a solid surface or not. Obviously, the water molecules are more strongly attracted to the glass molecules than they are to other water molecules, and thus water tends to rise along the glass surface. The opposite occurs for mercury, which causes the liquid surface near the glass wall to be suppressed (Fig. 2–37).

The magnitude of the capillary rise in a circular tube can be determined from a force balance on the cylindrical liquid column of height h in the tube (Fig. 2–38). The bottom of the liquid column is at the same level as the free surface of the reservoir, and thus the pressure there must be atmospheric pressure. This balances the atmospheric pressure acting at the top surface of the liquid column, and thus these two effects cancel each other. The weight of the liquid column is approximately

$$W = mg = \rho \mathcal{V} g = \rho g(\pi R^2 h)$$

Equating the vertical component of the surface tension force to the weight gives

$$W = F_{\text{surface}} \ \rightarrow \ \rho g(\pi R^2 h) = 2\pi R \sigma_s \cos \phi$$

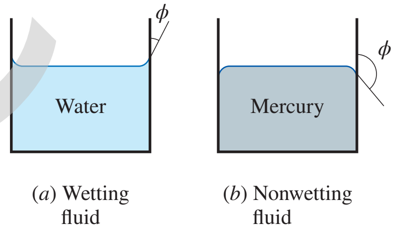


FIGURE 2–35
The contact angle for wetting and nonwetting fluids.

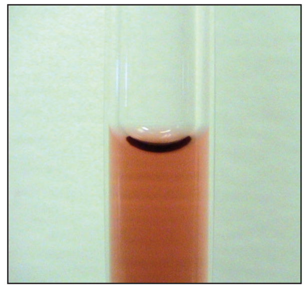


FIGURE 2–36
The meniscus of colored water in a 4-mm-inner-diameter glass tube. Note that the edge of the meniscus meets the wall of the capillary tube at a very small contact angle.

Photo by Gabrielle Trembley, Pennsylvania State University. Used by permission.

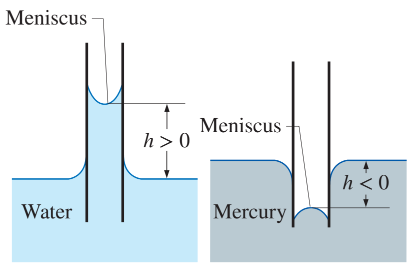


FIGURE 2–37
The capillary rise of water and the capillary fall of mercury in a small-diameter glass tube.

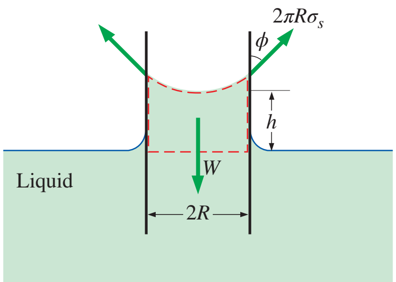


FIGURE 2–38
The forces acting on a liquid column that has risen in a tube due to the capillary effect.

Solving for h gives the capillary rise to be

Capillary rise:
$$h = \frac{2\sigma_s}{\rho g R} \cos \phi \qquad (R = \text{constant}) \tag{2–42}$$

This relation is also valid for nonwetting liquids (such as mercury in glass) and gives the capillary drop. In this case $\phi > 90°$ and thus $\cos \phi < 0$, which makes h negative. Therefore, a negative value of capillary rise corresponds to a capillary drop (Fig. 2–37).

Note that the capillary rise is inversely proportional to the radius of the tube. Therefore, the thinner the tube is, the greater the rise (or fall) of the liquid in the tube. In practice, the capillary effect for water is usually negligible in tubes whose diameter is greater than 1 cm. When pressure measurements are made using manometers and barometers, it is important to use sufficiently large tubes to minimize the capillary effect. The capillary rise is also inversely proportional to the density of the liquid, as expected. Therefore, in general, lighter liquids experience greater capillary rises. Finally, it should be kept in mind that Eq. 2–42 is derived for constant-diameter tubes and should not be used for tubes of variable cross section.

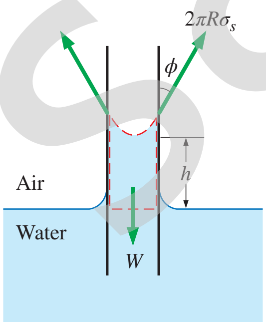


FIGURE 2–39
Schematic for Example 2–6.

EXAMPLE 2–6 **The Capillary Rise of Water in a Tube**

A 0.6-mm-diameter glass tube is inserted into water at 20°C in a cup. Determine the capillary rise of water in the tube (Fig. 2–39).

SOLUTION The rise of water in a slender tube as a result of the capillary effect is to be determined.

Assumptions **1** There are no impurities in the water and no contamination on the surfaces of the glass tube. **2** The experiment is conducted in atmospheric air.

Properties The surface tension of water at 20°C is 0.073 N/m (Table 2–4). The contact angle of water with glass is approximately 0° (from preceding text). We take the density of liquid water to be 1000 kg/m^3.

Analysis The capillary rise is determined directly from Eq. 2–42 by substituting the given values, yielding

$$h = \frac{2\sigma_s}{\rho g R} \cos \phi = \frac{2(0.073 \text{ N/m})}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.3 \times 10^{-3} \text{ m})} (\cos 0°) \left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N}} \right)$$
$$= 0.050 \text{ m} = \mathbf{5.0 \text{ cm}}$$

Therefore, water rises in the tube 5 cm above the liquid level in the cup.

Discussion Note that if the tube diameter were 1 cm, the capillary rise would be 0.3 mm, which is hardly noticeable to the eye. Actually, the capillary rise in a large-diameter tube occurs only at the rim. The center does not rise at all. Therefore, the capillary effect can be ignored for large-diameter tubes.

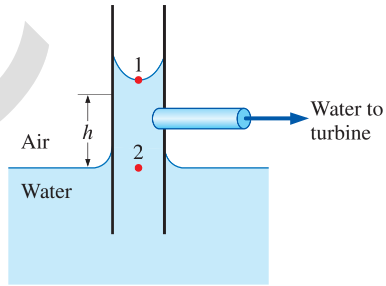


FIGURE 2–40
Schematic for Example 2–7.

EXAMPLE 2–7 Using Capillary Rise to Generate Power in a Hydraulic Turbine

Reconsider Example 2–6. Realizing that water rises by 5 cm under the influence of surface tension without requiring any energy input from an external source, a person conceives the idea that power can be generated by drilling a hole in the tube just below the water level and feeding the water spilling out of the tube into a turbine (Fig. 2–40). The person takes this idea even further by suggesting that a series of tube banks can be used for this purpose and cascading can be incorporated to achieve practically feasible flow rates and elevation differences. Determine if this idea has any merit.

SOLUTION Water that rises in tubes under the influence of the capillary effect is to be used to generate power by feeding it into a turbine. The validity of this suggestion is to be evaluated.

Analysis The proposed system may appear like a stroke of genius, since the commonly used hydroelectric power plants generate electric power by simply capturing the potential energy of elevated water, and the capillary rise provides the mechanism to raise the water to any desired height without requiring any energy input.

When viewed from a thermodynamic point of view, the proposed system immediately can be labeled as a perpetual motion machine (PMM) since it continuously generates electric power without requiring any energy input. That is, the proposed system creates energy, which is a clear violation of the first law of thermodynamics or the conservation of energy principle, and it does not warrant any further consideration. But the fundamental principle of conservation of energy did not stop many from dreaming about being the first to prove nature wrong, and to come up with a trick to permanently solve the world's energy problems. Therefore, the impossibility of the proposed system should be demonstrated.

As you may recall from your physics courses (also to be discussed in the next chapter), the pressure in a static fluid varies in the vertical direction only and increases with increasing depth linearly. Then the pressure difference across the 5-cm-high water column in the tube becomes

$$\begin{aligned}\Delta P_{\text{water column in tube}} &= P_2 - P_1 = \rho_{\text{water}} g h \\ &= (1000\ \text{kg/m}^2)(9.81\ \text{m/s}^2)(0.05\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg·m/s}^2}\right) \\ &= 0.49\ \text{kN/m}^2\ (\approx 0.005\ \text{atm})\end{aligned}$$

That is, the pressure at the top of the water column in the tube is 0.005 atm *less* than the pressure at the bottom. Noting that the pressure at the bottom of the water column is atmospheric pressure (since it is at the same horizontal line as the water surface in the cup) the pressure anywhere in the tube is below atmospheric pressure with the difference reaching 0.005 atm at the top. Therefore, if a hole were drilled at some elevation in the tube, the top of the meniscus would fall until its elevation was the same as that of the hole.

Discussion The water column in the tube is motionless, and thus, there cannot be any unbalanced force acting on it (zero net force). The force due to the pressure difference across the meniscus between the atmospheric air and the water at the top of water column is balanced by the surface tension. If this surface-tension force were to disappear, the water in the tube would drop down under the influence of atmospheric pressure to the level of the free surface in the tube.