

 NINTH Edition

# **BUSINESS MATHEMATICS** in CANADA

F. ERNEST JEROME  
TRACY WORSWICK

**NINTH EDITION**

# **BUSINESS MATHEMATICS** in CANADA

**F. Ernest Jerome**

**Tracy Worswick**  
Conestoga College

**Mc  
Graw  
Hill**  
Education





## **BUSINESS MATHEMATICS IN CANADA**

### **Ninth Edition**

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ISBN-13: 978-1-25-926582-2

ISBN-10: 1-25-926582-X

1 2 3 4 5 6 7 8 9 0 TCP 1 9 8 7

Printed and bound in Canada.

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Cover Design: Lightbox Visual Communications Inc.

Cover Images: Image Source/Alamy Stock Photo, Polygraphus/Getty Images

Interior Design: Michelle Losier

Composition: Aptara, Inc.

Page Layout: Tom Dart

Printer: Transcontinental Printing Group



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# Preface

Most business administration programs in Canadian colleges include an introductory course in business mathematics or mathematics of finance. *Business Mathematics in Canada* is intended for use in such courses. The text's primary objective is to support the learning of mathematics (other than statistics) needed to succeed in fields such as accounting, finance, management, marketing, personal financial planning, and business information systems.

This book may be adapted to either a one- or a two-semester course in business mathematics. It is suitable for courses that emphasize either an algebraic approach or a pre-programmed financial calculator approach to compound interest problems. (Optional spreadsheet templates provide a third alternative in many areas for students who have a basic familiarity with Microsoft Excel software.) Both algebraic solutions and financial calculator solutions are presented in most example problems for compound interest topics.

## NEW IN THE NINTH EDITION

The ninth edition contains numerous changes reflecting input from faculty across the country, through reviews as well as invaluable suggestions from users of the eighth edition.

**New and Updated Examples** The worked examples now include **20** new problems with full solutions; another **50** examples have been updated to reflect current rates and prices.

**New Calculator-Free Problems and New and Updated Exercises** Calculator-free problems are new in Chapters 1 through 4 and provide exercises where students can focus primarily on the problem-solving process. The Exercises contain **100** new problems, and another **100** problems have been updated to reflect current rates and prices.

**Updated Tables and Charts** The exposition, tables, and example problems incorporate the most recent data at the time of writing.

**Solving Two Equations and Two Unknowns** has been moved to Chapter 5 with Applications of Linear Equations.

**Return on Investments** has been moved to Chapter 2 with the Basic Percentage Problem and Percent Change.

**New Tips and Traps** The ninth edition features **7** new Tips and Traps—on “Common Rounding Errors” and “Determining the Weight Factor” in Chapter 1; “The Immediate-Left Trick” and “Excel Applications in Business Mathematics” in Chapter 2; “How Low Does It Go?” and “Always Use Given Information” in Chapter 3; and “‘Cost’ Is Not the Selling Price” in Chapter 4.

**Revised Cases and Points of Interest** These features have been revised and refreshed to incorporate recent developments and time-sensitive data. The ninth edition features **7** new Points of Interest: “Following the Ups and Downs of Gas Prices in Canada” in Chapter 2, which looks at percent increases and decreases at the gas pumps; “Markup versus Margin” in Chapter 4, which examines the main differences between the two concepts; “Which Years Are Leap Years?” in Chapter 6, which explains various techniques for determining leap years; “Got a Million Dollar Talent?” in Chapter 10, which investigates the real time value of a \$1M prize; “What Is Your Net Worth?” in Chapter 11, which examines this measure of personal wealth and provides a template for students to calculate their own net worth; “Rent to Own Real Estate” in Chapter 12, which explains the basics of rent to own real estate deals; and “No-Money-Down Mortgages Are No More” in Chapter 14, which summarizes the changes in Canadian mortgage rules over the past 10 years.

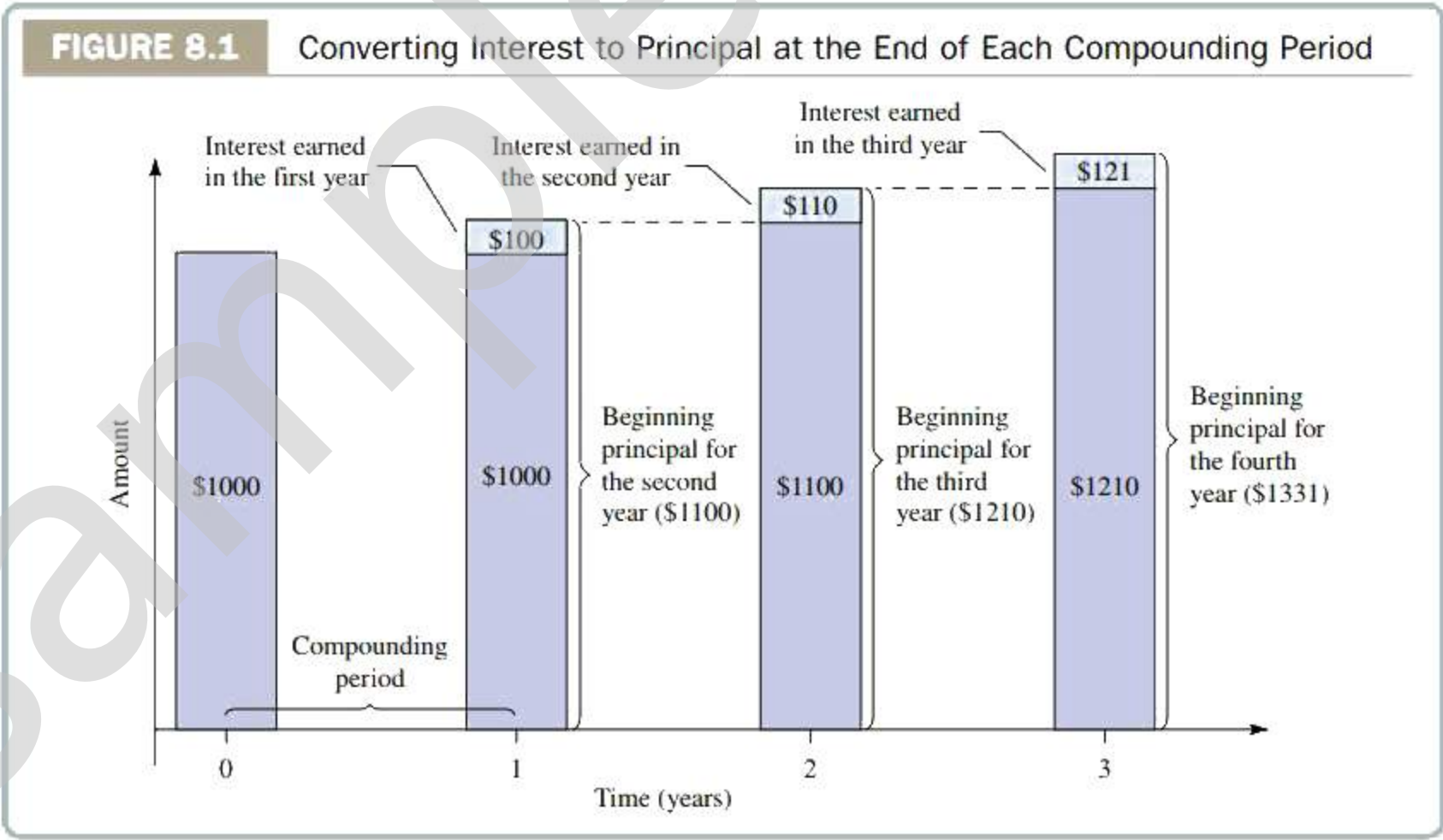


# PEDAGOGICAL FEATURES

**Canadian Applications** Throughout the exposition, Example problems, Exercise problems, and Points of Interest, the book presents a wide range of applications of mathematics in Canadian business and finance. Every effort has been made to reflect current practices. Real financial instruments and real economic data are frequently used.

**Wide Selection of Problems** Each section of a chapter is followed by a set of problems for applying and reinforcing the new material. The text contains over 2000 problems and concept questions. Questions are organized by “calculator-free,” “basic,” “intermediate,” and “advanced.” Considerable effort has been made to create problems that are instructive, practical, realistic, and interesting.

**Graphs and Diagrams** This text makes extensive use of graphs, diagrams, and interactive charts.



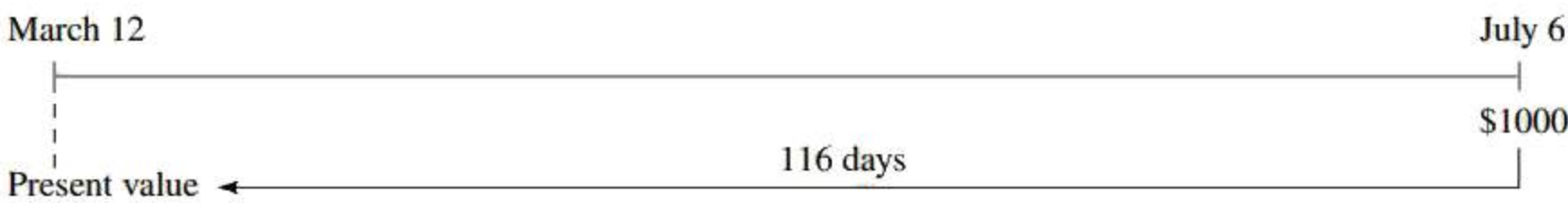
**Solved Example Problems** These examples provide detailed illustrations and applications of concepts in a step-by-step format.

**EXAMPLE 6.4B** CALCULATING AN EQUIVALENT PAYMENT AT AN EARLIER DATE

What is the value of a payment to be received on March 12 if it is economically equivalent to a \$1000 payment originally due on the subsequent July 6, if money is worth 6.8% per year?

**SOLUTION**

Since we want an equivalent payment at an earlier date, we should calculate the *present* value of \$1000 on March 12.



The number of days in the interval is

$$20 \text{ (for March)} + 30 + 31 + 30 + 5 \text{ (for July)} = 116$$

Substituting  $S = \$1000$ ,  $r = 6.8\%$ , and  $t = \frac{116}{365}$  into Formula (6-3), we obtain

$$P = \frac{S}{1 + rt} = \frac{\$1000}{1 + 0.068\left(\frac{116}{365}\right)} = \$978.85$$

**\$978.85 on March 12 is equivalent to \$1000 on the subsequent July 6.**



**Tips and Traps** Boxed elements inserted at appropriate points draw the student's attention to simplifications, pitfalls, shortcuts, calculator procedures, and common errors.

**TIP**

**Avoid Fractions in Equations**

You will avoid fractions in your equations, as seen in [Example 2.5A](#) part (b), if you let the variable  $x$  represent the smallest quantity in a relationship with two or more unknown quantities.

**TRAP**

**Percent Changes Are Not Additive**

As the preceding example demonstrates, the overall percent change for a series of intervals cannot be obtained simply by adding the percent changes for the individual intervals. The reason is that the initial value for the percent change calculation is different in each interval.

**Point of Interest Boxes** Most chapters contain two or three intriguing illustrations of the application or misapplication of mathematics to business and personal finance. See the Points of Interest list that follows the table of contents.

**POINT OF INTEREST**

**The Price-to-Performance Ratio**

People in the computer industry have recognized that the price-to-performance ratio comparison presented in advertisements can actually provide a reason *not* to purchase the advertised computer. Think about what, at a basic mathematical level, will make the price-to-performance ratio high. A large numerator (*high* price) and/or a small denominator (*low* performance) will make the ratio high. Therefore, a higher ratio represents a *lower* “bang for your buck.”

The price-to-performance ratio can also be used to track changes in products over time. Computers in particular have a constantly fluctuating price-to-performance ratio. Over time the price of computers has dropped dramatically, making them more accessible to the average consumer while processing power has improved significantly.

**App 4 That** Each boxed feature provides key words that are searchable on a smart device to find free and paid apps relevant to the topic under discussion.

**APP 4 THAT**

To find a quick way either to solve an equation or to gain further understanding on how to solve an equation in your homework problems, search the App Store on your tablet, smartphone, or smart watch using the key words **EQUATION SOLVER**.

You will find many free and paid apps that generate step-by-step solutions to guide you through the solving process either by typing in the equation or by taking a photo of the equation to be solved.



**Calculator Callout Boxes** Many compound interest calculations can be performed using a calculator’s financial functions. In the solutions for Example problems, we employ callout boxes: (1) to provide a clear visual indication of the algebraic computations that may be executed using the calculator’s pre-programmed financial functions; and (2) to present the keystroke operations for employing the financial functions.

**EXAMPLE 10.2A** THE FUTURE VALUE OF PERIODIC INVESTMENTS

Heinz has been contributing \$300 at the end of each month for the past 15 months to a savings plan that earns 6% compounded monthly. What amount will he have one year from now if he continues with the plan?

**SOLUTION**  
The total amount will be the future value of  $n = 15 + 12 = 27$  contributions of  $PMT = \$300$  each. Payments and compounding both occur at one-month intervals. Therefore, the payments form an ordinary simple annuity having  $i = \frac{6\%}{12} = 0.5\%$  per month.

$$\begin{aligned} FV &= PMT \left[ \frac{(1+i)^n - 1}{i} \right] \\ &= \$300 \left[ \frac{(1.005)^{27} - 1}{0.005} \right] \\ &= \$300 \left( \frac{1.14415185 - 1}{0.005} \right) \\ &= \$8649.11 \end{aligned}$$

One year from now, Heinz will have \$8649.11 in the plan.

6 I/Y

P/Y 12 ENTER

(making C/Y = P/Y = 12)

27 N

0 PV

300 +/- PMT

CPT FV

Ans: 8649.11


**Highlighted Concepts** Throughout the book, statements of key concepts are highlighted, signalling to students the importance of the concept or principle.

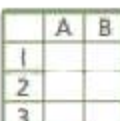
These ideas are so important and of such wide application in finance that they are formally embodied in the Valuation Principle.

**Valuation Principle**  
The fair market value of an investment is the sum of the present values of the cash flows expected from the investment. The discount rate used in the present value calculations should be the prevailing market-determined rate of return on this type of investment.

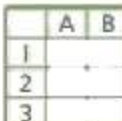
If the expected cash flows are received as forecast, the investor's actual rate of return on the amount invested will be precisely the discount rate used in the fair market value calculation.

**Spreadsheet Templates** Example problems and exercises indicated with a spreadsheet icon will direct students to an **optional** Microsoft Excel spreadsheet available on Connect. Each spreadsheet either demonstrates a solution of the example problem solved previously by algebraic or financial calculator methods or provides an alternative platform for solving the exercise problems. The spreadsheet is based on a pre-labelled and pre-formatted template.

 **TIP**

**Excel Applications in Business Mathematics**  
 If you have not covered **Section 2.8** in your course, go back and read the TIP box titled “Excel Applications in Business Mathematics” located in that section. It explains how Connect provides spreadsheet applications as an optional feature.

**EXAMPLE 4.1E** CALCULATING THE NET PRICE AFTER MULTIPLE DISCOUNTS

 WGW Manufacturing and Ace Clothing both produce basic work shirts that are very similar in quality and popularity. Both manufacturers quote a list price of \$46.00 for the shirt. WGW offers a regular trade discount of 25% plus an additional volume discount of 10% on orders of at least 1000 shirts. Ace offers a standard discount of 30% and a further 5% discount on orders exceeding 500 shirts. Which source will give the lower net price on an order for 1000 shirts? How much lower per shirt?

**SOLUTION**  
Given: For WGW,  $L = \$46.00$ ,  $d_1 = 25\%$ ,  $d_2 = 10\%$   
For Ace,  $L = \$46.00$ ,  $d_1 = 30\%$ ,  $d_2 = 5\%$   
The net price per shirt from WGW is



**Cases** Some chapters include a case study in the end-of-chapter material. These cases usually call on concepts and skills from previous chapters as well as the current chapter.

CASE Calculations for an Investment Portfolio

One year ago, Jasmin and Derek opened investment accounts with a discount broker. In their C\$ account, they purchased 300 Bank of Montreal (BMO) shares at C\$54.20 per share and six Government of Canada bonds (GoCs) at C\$1063 per bond. In their US\$ account, they purchased 100 shares of International Business Machines (IBM) at US\$125.50 per share and 200 shares of General Electric (GE) at US\$18.57 per share. The exchange rate on the date of the purchases was C\$1.00 = US\$0.935.

The income received from the securities during the year and their current prices are listed in the following table.

	Number owned	Income received	Current price
BMO	300 shares	C\$3.20 per share	C\$58.15
GoCs	6 bonds	C\$70.00 per bond	C\$1021.50
IBM	100 shares	US\$2.50 per share	US\$132.25
GE	200 shares	US\$0.50 per share	US\$20.38

The current exchange rate is C\$1.00 = US\$0.952

QUESTIONS

1. What total amount in C\$ was initially invested in the portfolio?
2. What is the C\$ equivalent of the total income received during the year? (For converting the US\$ income to C\$, use the average of the beginning and ending exchange rates for the year.)
3. What is the current total value in C\$ of the securities (not including income received)?
4. Including both income and change in value of the securities, what was the percentage increase in the value of Jasmin's and Derek's portfolios during the year?

**Interactive Charts** Through the online Connect platform, students can access interactive charts. Various Exercise problems (flagged by a “Connect” icon) invite the student to undertake an activity using an interactive chart. The student can change key variables and observe a graphic representation of the effect on the dependent variable (such as break-even point, present value, future value, market value, interest paid, amortization period, etc.).

**End-of-Chapter Problems** Each chapter ends with a comprehensive set of Review Problems covering the full range of topics and applications in the chapter.

**Concept Questions** Concept Questions are presented at the end of many sections. These questions exercise students’ intuition and test their understanding of concepts and principles.

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- Post your own resources for students to use.

### Instructor Resources

- **Instructor's Solutions Manual.** Prepared by the author, with a technical review by Sarah Chan.
- **Computerized Test Bank.** Prepared by Sarah Chan.
- **Microsoft® PowerPoint® Lecture Slides.** Prepared by Rob Sorenson of Camosun College.



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For more information, please visit us online: <http://www.mheducation.ca/he/solutions>

## ACKNOWLEDGMENTS

The ninth edition represents the compound future value of contributions from many dedicated educators over the previous editions. A new debt of gratitude is owed to professors who participated in a review or worked on supplements for this edition. Their thoughtful comments and suggestions have made this a better text.

Our reviewers:

Lisa MacKay	<i>SAIT Polytechnic</i>
Erica Colter	<i>NSCC</i>
Sylvia Leskiw	<i>MacEwan University</i>
Pamela Quon	<i>Athabasca University</i>
Deborah Sauer	<i>Capilano University</i>
John Molendyk	<i>Capilano University</i>
James Hebert	<i>Red River College</i>
Asifa Aamir	<i>UOIT</i>
Rob Sorensen	<i>Camosun College</i>

I am especially indebted to those who have served as a Technical Reviewer or who have been instrumental in the preparation of supplemental resources for the book. Thanks in this regard go to Sarah Chan and Rob Sorenson.

I wish to thank the staff at McGraw-Hill Ryerson for their ongoing support, professionalism, and guidance: Sara Braithwaite (Product Manager), Erin Catto (Product Developer), Joanne Limebeer (Supervising Editor), Karen Fozard (Portfolio and Program Manager), and all others involved in the development and production of this edition.

I have been fortunate during the writing of this ninth edition to be teaching many of the topics at the same time as their writing. I wish to thank all of my students, past and present, for their valuable feedback, providing insight into the content of this edition through their eyes.

Finally, I send my love and thanks to my husband Mike and family members Leslie, John, Kasey, and Jeff, who are always there to support and encourage me and for providing a lot of take-out meals when deadlines were near. Thanks also go to Chloe and Lola for tirelessly sleeping under my desk (at the risk of being rolled over by my office chair) as this ninth edition came to life.

**Tracy Worswick**





## Chapter 1

# Review and Applications of Basic Mathematics

### CHAPTER OUTLINE

- 1.1** Order of Operations
- 1.2** Fractions, Decimals, and Percents
- \*1.3** Payroll
- 1.4** Simple and Weighted Averages
- \*1.5** Taxes

**Appendix 1A:** The Texas Instruments BA II PLUS Format Settings

(Sections and chapters preceded by an asterisk\* may be omitted without loss of continuity.)

### LEARNING OBJECTIVES

*After completing this chapter, you will be able to:*

- L01** Perform arithmetic operations in their proper order
- L02** Convert fractions to their percent and decimal equivalents
- L03** Maintain the proper number of digits in calculations
- L04** Perform calculations using fractions, decimals, and percents
- L05** Calculate the gross earnings of employees paid a salary, an hourly wage, or commissions
- L06** Calculate the simple average or weighted average (as appropriate) of a set of values
- L07** Perform basic calculations for the Goods and Services Tax, Harmonized Sales Tax, provincial sales tax, and real property tax



**MATHEMATICS PLAYS A SIGNIFICANT ROLE** in business. Clients and employers now expect higher education and outstanding performance from all levels of employees that includes a significant level of mathematics competency.

Even though most routine calculations in business are done electronically, the mathematics and statistics you study in your business program are more widely expected and more highly valued in business than ever before. As a successful business graduate you must know which information is relevant, which analyses or calculations should be performed, how to interpret the results, and how to explain the outcome in terms your clients and colleagues can understand.

Naturally, a college course in business mathematics or statistics will cover a broader range of topics (often in greater depth) than you might need for a particular industry. This broader education opens more career options to you and provides a stronger set of mathematical skills for your chosen career.



**TIP**

### How to Succeed in Business Mathematics

Since various Business Mathematics and Mathematics of Finance courses start at different points in the book, this Tip appears at the beginning of each of the first four chapters. Connect has a guide entitled “How to Succeed in Business Mathematics.” Read its first two sections (A.1 and A.2) before you finish **Chapter 1**.

## 1.1 Order of Operations

**L01** When evaluating an expression such as

$$5 + 3^2 \times 2 - 4 \div 6$$

there is potential for confusion about the sequence of mathematical steps. Do we just perform the indicated operations in a strict left-to-right sequence called *chaining*, or is some other order intended? To eliminate any possible confusion, mathematicians have agreed on the algebraic operating system (AOS), which sets out the rules for the use of brackets and the order of mathematical operations. The rules are:

### Rules for Order of Operations

1. Perform operations within brackets (in the order of Steps 2, 3, and 4 below).
2. Evaluate the powers.<sup>1</sup>
3. Perform multiplication and division in order from left to right.
4. Perform addition and subtraction in order from left to right.



**TIP**

### Remembering the Order of Operations: BEDMAS

To help remember AOS, the order of operations, you can use the acronym “BEDMAS” representing the sequence: **B**rackets, **E**xponents, **D**ivision and **M**ultiplication, **A**ddition and **S**ubtraction.

<sup>1</sup> A power is a quantity such as  $3^2$  or  $5^3$  (which are shorthand methods for representing  $3 \times 3$  and  $5 \times 5 \times 5$ , respectively). Section 2.2 includes a review of powers and exponents.



**EXAMPLE 1.1A****EXERCISES ILLUSTRATING THE ORDER OF MATHEMATICAL OPERATIONS**

- a.  $30 - 6 \div 3 + 5$  Do division before subtraction and addition.  
 $= 30 - 2 + 5$   
 $= 33$
- b.  $(30 - 6) \div 3 + 5$  Do operations within brackets first; then do division before addition.  
 $= 24 \div 3 + 5$   
 $= 8 + 5$   
 $= 13$
- c.  $\frac{30 - 6}{3 + 5} = \frac{24}{8} = 3$  Brackets are implied in the numerator and the denominator.
- d.  $72 \div (3 \times 2) - 6$  Do operations within brackets first; then do division before subtraction.  
 $= 72 \div 6 - 6$   
 $= 12 - 6$   
 $= 6$
- e.  $72 \div (3 \times 2^2) - 6$  Do operations within brackets (the power before the multiplication); then do division before subtraction.  
 $= 72 \div (3 \times 4) - 6$   
 $= 72 \div 12 - 6$   
 $= 6 - 6$   
 $= 0$
- f.  $72 \div (3 \times 2)^2 - 6$  Do operations within brackets first, then the power, then divide, then subtract.  
 $= 72 \div 6^2 - 6$   
 $= 72 \div 36 - 6$   
 $= 2 - 6$   
 $= -4$
- g.  $4(2 - 5) - 4(5 - 2)$  Do operations within brackets first, then multiplication, then subtract.  
 $= 4(-3) - 4(3)$   
 $= -12 - 12$   
 $= -24$

**EXERCISE 1.1**

Answers to the odd-numbered problems are at the end of the book.

**CALCULATOR-FREE PROBLEMS**

Evaluate each of the following.

- |                                  |   |
|----------------------------------|---|
| a. $10 + 10 \times 0$            | b. $2 \times 2 + 4 - 8$                 |
| c. $(10 + 10) \times 0$          | d. $2 \times (2 + 4) - 8$               |
| e. $0 + 3 \times 3 - 3^2 + 10$   | f. $12 - 2 \times 5 + 2^2 \times 0$     |
| g. $0 + 3 \times 3 - (3^2 + 10)$ | h. $(12 - 2) \times (5 + 2^2) \times 0$ |
| i. $\frac{2^2 - 4}{(4 - 2)^2}$   | j. $\frac{(2 - 4)^2}{5 - 2^2}$          |



**BASIC PROBLEMS**

Evaluate each of the following. In Problems 17–22, evaluate the answers accurate to the cent.

1.  $20 - 4 \times 2 - 8$
2.  $18 \div 3 + 6 \times 2$
3.  $(20 - 4) \times 2 - 8$
4.  $18 \div (3 + 6) \times 2$
5.  $20 - (4 \times 2 - 8)$
6.  $(18 \div 3 + 6) \times 2$
7.  $54 - 36 \div 4 + 2^2$
8.  $(5 + 3)^2 - 3^2 \div 9 + 3$
9.  $(54 - 36) \div (4 + 2)^2$
10.  $5 + (3^2 - 3)^2 \div (9 + 3)$
11.  $\frac{8^2 - 4^2}{(4 - 2)^3}$
12.  $\frac{(8 - 4)^2}{4 - 2^3}$
13.  $3(6 + 4)^2 - 5(17 - 20)^2$
14.  $(4 \times 3 - 2)^2 \div (4 - 3 \times 2^2)$
15.  $[(20 + 8 \times 5) - 7 \times (-3)] \div 9$
16.  $5[19 + (5^2 - 16)^2]^2$
17.  $\$100(1 + 0.06 \times \frac{45}{365})$
18.  $\frac{\$200}{1 + 0.09 \times \frac{4}{12}}$
19.  $\frac{\$500}{(1 + 0.05)^2}$
20.  $\$1000(1 + 0.02)^3$
21.  $\$100 \left[ \frac{(1 + 0.04)^2 - 1}{0.04} \right]$
22.  $\$300 \left[ \frac{1 - \frac{1}{(1 + 0.03)^2}}{0.03} \right]$

**1.2** Fractions, Decimals, and Percents**Definitions**

In the fraction  $\frac{3}{4}$ , the upper number (3) is called the **numerator** (or dividend) and the lower number (4) is the **denominator** (or divisor). In a **proper fraction**, the numerator is smaller than the denominator. An **improper fraction** has a numerator that is larger than the denominator. A **mixed number** contains a whole number plus a fraction. **Equivalent fractions** are fractions that are equal in value (even though their respective numerators and denominators differ). An equivalent fraction can be created by multiplying or dividing both the numerator and the denominator by the same number.

**EXAMPLE 1.2A****EXAMPLES OF TYPES OF FRACTIONS**

- a.  $\frac{6}{13}$  is a proper fraction.
- b.  $\frac{17}{13}$  is an improper fraction.
- c.  $2\frac{4}{13}$  is a mixed number.
- d.  $\frac{5}{13}$ ,  $\frac{10}{26}$ ,  $\frac{15}{39}$ , and  $\frac{20}{52}$  are equivalent fractions. Note that the second, third, and fourth fractions may be obtained by multiplying *both* the numerator and the denominator of the first fraction by 2, 3, and 4, respectively.



**EXAMPLE 1.2B** CALCULATING AN EQUIVALENT FRACTION

Find the missing numbers that make the following three fractions equivalent.

$$\frac{7}{12} = \frac{56}{?} = \frac{?}{300}$$

**SOLUTION**

To create a fraction equivalent to  $\frac{7}{12}$ , both the numerator and the denominator must be multiplied by the same number. To obtain 56 in the numerator of the second equivalent fraction, 7 was multiplied by 8. Hence, the denominator must also be multiplied by 8. Therefore,

$$\frac{7}{12} = \frac{7 \times 8}{12 \times 8} = \frac{56}{96}$$

To obtain the denominator (300) in the third equivalent fraction, 12 was multiplied by  $\frac{300}{12} = 25$ .

The numerator must also be multiplied by 25. Hence, the equivalent fraction is

$$\frac{7 \times 25}{12 \times 25} = \frac{175}{300}$$

In summary,

$$\frac{7}{12} = \frac{56}{96} = \frac{175}{300}$$

**Decimal and Percent Equivalents**

**L02** In the fraction  $\frac{3}{4}$ , the denominator indicates the total number of parts or pieces and the numerator shows how many of the parts we are considering. In other words,  $\frac{3}{4}$  is 3 of 4 parts.

The *decimal equivalent* value of a fraction is obtained by dividing the numerator by the denominator. The fraction  $\frac{3}{4}$  then becomes the decimal equivalent 0.75, indicating 0.75 parts of one whole piece.

To express the fraction in *percent equivalent* form, multiply the decimal equivalent by 100 (shift the decimal point two places to the right) and add the % symbol indicating parts of 100. The fraction  $\frac{3}{4}$  then becomes 75%, indicating 75 parts of 100 parts.

**EXAMPLE 1.2C** FINDING THE DECIMAL AND PERCENT EQUIVALENTS OF FRACTIONS AND MIXED NUMBERS

Convert each of the following fractions and mixed numbers to its decimal equivalent and percent equivalent values.

a.  $\frac{2}{5} = 0.4 = 40\%$

b.  $\frac{5}{2} = 2.5 = 250\%$

c.  $2\frac{3}{4} = 2.75 = 275\%$

d.  $\frac{5}{8} = 0.625 = 62.5\%$

e.  $1\frac{3}{16} = 1.1875 = 118.75\%$

f.  $\frac{3}{1500} = 0.002 = 0.2\%$

**TIP****Adding or Subtracting Fractions**

To add or subtract any but the simplest of fractions, the easiest approach is to first convert each fraction to its decimal equivalent value. Then add or subtract the decimal equivalents as required.

For example,  $\frac{5}{12} + \frac{23}{365} = 0.41667 + 0.06301 = 0.4797$  to four-figure accuracy.



## Rounding of Decimal and Percent Equivalents

**L03** For some fractions, the decimal equivalent has an endless series of digits. Such a number is called a *nonterminating decimal*. In some cases a nonterminating decimal contains a repeating digit or a repeating group of digits. This particular type of nonterminating decimal is referred to as a *repeating decimal*. A shorthand notation for repeating decimals is to place a horizontal bar over the first occurrence of the repeating digit or group of digits. For example,

$$\frac{2}{9} = 0.222222 = 0.\overline{2} \quad \text{and} \quad 2\frac{4}{11} = 2.36363636 = 2.\overline{36}$$

When a nonterminating decimal or its percent equivalent is used in a calculation, the question arises: How many figures or digits should be retained? The following rules provide sufficient accuracy for the vast majority of our calculations.

### Rules for Rounding Numbers

1. In intermediate results, keep one more figure than the number of figures required in the final result. (When counting figures for the purpose of rounding, do not count leading zeros used only to properly position the decimal point.)<sup>2</sup>
2. If the first digit dropped is 5 or greater, increase the last retained digit by 1.
3. If the first digit dropped is less than 5, leave the last retained digit unchanged.

Suppose, for example, the answer to a calculation is expected to be a few hundred dollars and you want the answer accurate to the cent. In other words, you require five-figure accuracy in your answer. To achieve this accuracy, the first rule says you should retain (at least) six figures in values used in the calculations. The rule also applies to intermediate results that you carry forward to subsequent calculations. The consequence of rounding can be stated in another way—if, for example, you use a number rounded to four figures in your calculations, you can expect only three-figure accuracy in your final answer.

### EXAMPLE 1.2D

### FRACTIONS HAVING REPEATING DECIMAL EQUIVALENTS

Convert each of the following fractions to its decimal equivalent value expressed in the repeating decimal notation.

a.  $\frac{2}{3} = 0.6666\ldots = 0.\overline{6}$

b.  $\frac{14}{9} = 1.555\ldots = 1.\overline{5}$

c.  $6\frac{1}{12} = 6.08333\ldots = 6.08\overline{3}$

d.  $3\frac{2}{11} = 3.181818\ldots = 3.\overline{18}$

e.  $5\frac{2}{27} = 5.074074\ldots = 5.\overline{074}$

f.  $\frac{5}{7} = 0.714285714285\ldots = 0.\overline{714285}$

<sup>2</sup> The following example illustrates the reasoning behind this instruction. A length of 6 mm is neither more nor less precise than a length of 0.006 m. (Recall that there are 1000 mm in 1 m.) The leading zeros in 0.006 m do not add precision to the measurement. They are inserted to properly position the decimal point. Both measurements have one-figure accuracy. Contrast this case with measurements of 1007 mm and 1.007 m. Here each zero comes from a decision about *what* the digit should be (rather than *where* the decimal point should be). These measurements both have four-figure accuracy. This rule applies to the total number of figures (other than leading zeros) in a value. It does not apply to the number of *decimal* places.



**EXAMPLE 1.2E**      **CALCULATING AND ROUNDING THE DECIMAL EQUIVALENTS OF FRACTIONS**

Convert each of the following fractions and mixed numbers to its decimal equivalent value rounded to four-figure accuracy.

- a.  $\frac{2}{3} = 0.6667$                       b.  $6\frac{1}{12} = 6.083$                       c.  $\frac{173}{11} = 15.73$   
 d.  $\frac{2}{1071} = 0.001867$                       e.  $\frac{17,816}{3} = 5939$



**TRAP**

**Common Rounding Errors**

The following examples illustrate two of the most common rounding errors. Each example requires rounding to two decimal places or three significant digits:

Example 1:  $2.4449 = 2.445 = 2.45$  This is NOT correct.

For rounding to two decimal places, you only need to consider the third decimal place to apply Rule 3.

Here, continuous rounding has been applied starting with the first digit that is 5 or larger. The correct answer is 2.44.

Example 2:  $2.992 = 2.99 = 3.00$  This is NOT correct.

The value is originally rounded correctly using Rule 3 but then rounded again applying Rule 2.

Round one time only to achieve the required number of significant digits. The correct answer is 2.99.

**EXAMPLE 1.2F**      **DEMONSTRATING THE CONSEQUENCES OF TOO MUCH ROUNDING**

Accurate to the cent, evaluate

$$\$140\left(1 + 0.11 \times \frac{113}{365}\right) + \$74\left(1 + 0.09 \times \frac{276}{365}\right)$$

**SOLUTION**

If you want five-figure accuracy in your answer, you cannot round to fewer than six figures *at any stage* of the calculations. The following table illustrates how too much rounding can result in an inaccurate answer.

If we first evaluate the contents of the brackets before rounding, we obtain:  $\$140(1.0340548) + \$74(1.0680548)$

6-Figure Accuracy	5-Figure Accuracy	3-Figure Accuracy
$\$140(1.03405) + \$74(1.06805)$	$\$140(1.0341) + \$74(1.0681)$	$\$140(1.03) + \$74(1.07)$
$= \$144.767 + \$79.0357$	$= \$144.774 + \$79.0394$	$= \$144.20 + \$79.18$
$= \$223.8027$	$= \$223.8134$	$= \$223.38$
$= \$223.80$ (rounded to the cent)	$= \$223.81$ (rounded to the cent)	
Correct answer	\$0.01 larger than correct answer	\$0.42 smaller than correct answer

One more point is worth noting. Consider the first column, where you properly maintained six-figure accuracy. That is,

$$\$140(1.03405) + \$74(1.06805) = \$144.767 + \$79.0357$$

Suppose you round the two amounts on the right side to the nearest cent *before* you add them. The sum is then

$$\$144.77 + \$79.04 = \$223.81$$

which is \$0.01 larger than the correct answer. The error arises because, just at the final addition, you failed to maintain six-figure accuracy (to ensure five-figure accuracy in the final answer).



**TIP****Optimal Use of Your Calculator**

Whenever possible, use your calculator's memory registers to save intermediate results. This will save time and reduce keystroke errors during data re-entry. It also virtually eliminates the introduction of rounding errors, since most calculators internally retain two or three more figures than are shown in the display. **Example 1.2G** illustrates this approach.

**EXAMPLE 1.2G****OPTIMAL USE OF YOUR CALCULATOR**

We will again evaluate (accurate to the cent) the same expression as in **Example 1.2F**,

$$\$140\left(1 + 0.11 \times \frac{113}{365}\right) + \$74\left(1 + 0.09 \times \frac{276}{365}\right)$$

This time we will use our financial calculator in a way that (1) avoids manual re-entry of intermediate results, and (2) maintains maximum precision by avoiding rounding (other than rounding imposed by the inherent limitations of the calculator).

**SOLUTION**

We assume the Texas Instruments BA II PLUS calculator is set for a floating-decimal format and for the algebraic operating system (AOS) calculation method. (Refer to **Appendix 1A** for instructions on making these settings.) In the AOS mode, we can enter numbers, brackets, and operations in the same left-to-right sequence as they are written. The calculator performs the calculations according to the proper order of operations.

$$\begin{array}{cc} 140 & \times & ( & 1 & + & 0.11 & \times & 113 & \div & 365 & ) & + & 74 & \times & ( & 1 & + & 0.09 & \times & 276 & \div & 365 & ) & = & 223.80 \end{array}$$

The result is \$223.80.

You see that it is possible to evaluate quite complex expressions without writing down intermediate results. However, if someone is going to read and readily understand your solution, you should present enough detail and intermediate results to reveal the steps in your solution.

**TIP****Using a Calculator's Power Function**

Use the following sequence of keystrokes to evaluate  $1.62^5$  with the power function key  $y^x$ .

$$1.62 \quad y^x \quad 5 \quad =$$

If the symbol  $y^x$  sits above a calculator key (rather than on it), the power function is the secondary function of the key. The keystroke sequence is then

$$1.62 \quad 2^{\text{nd}} \quad y^x \quad 5 \quad =$$

The answer to seven-figure accuracy is 11.15771.

**Example 1.2H** uses this feature.



## Evaluating Complex Fractions

A **complex fraction** is a fraction containing one or more other fractions in its numerator or denominator. In simplifying complex fractions, particular attention should be paid to the correct order of mathematical operations as discussed in [Section 1.1](#).

### EXAMPLE 1.2H EVALUATING COMPLEX FRACTIONS

Evaluate each of the following complex fractions accurate to the cent.

a.  $\frac{\$425}{\left(1 + \frac{0.09}{12}\right)^{24}}$

b.  $\frac{\$1265\left(1 + 0.115 \times \frac{87}{365}\right)}{1 + 0.125 \times \frac{43}{365}}$

c.  $\frac{\$1}{1 + 0.025 \times \frac{5}{12}} + \frac{\$1}{1 + 0.04 \times \frac{2}{12}}$

#### SOLUTION

We assume the Texas Instruments BA II PLUS calculator is set for a floating-decimal format and for the algebraic operating system (AOS) calculation method. Refer to [Appendix 1A](#) for instructions on making these settings.

a. 425  $\div$  ( 1 + 0.09  $\div$  12 )  $y^x$  24 = 355.23

The result is \$355.23.

b. One-step method where additional brackets must be used for the denominator:

1265  $\times$  ( 1 + 0.115  $\times$  87  $\div$  365 )  $\div$  ( 1 + 0.125  $\times$  43  $\div$  365 ) = 1280.81

Two-step method using the calculator's memories to store intermediate answers:

$1265 \times (1 + 0.115 \times 87 \div 365) = 1299.674863$  STO 1

$1 + 0.125 \times 43 \div 365 = 1.014726027$  STO 2

RCL 1  $\div$  RCL 2 = \$1280.81

The result is \$1280.81

c.  $1 + 0.025 \times 5 \div 12 = 1.0104166667$  1/X (0.989690722) STO 1

$1 + 0.04 \times 2 \div 12 = 1.006666667$  1/X (0.993377483) STO 2

RCL 1 + RCL 2 = \$1.98

The result is \$1.98.

## Calculating Percent of a Number

**L04** Calculating the percent of a number is one of the most common calculations in business. To find the percent of a number, convert the percent to its decimal equivalent by dividing the percent by 100 (shifting the decimal point two places to the left) and then multiplying by the number.



**EXAMPLE 1.2I**

- a. What is 22% of \$185?                      b. What is  $40\frac{1}{4}\%$  of \$140.25?  
 c. How much is  $0.08\bar{3}\%$  of \$5000?        d. How much is 140% of \$50?

**SOLUTION**

- a. The question asks us to calculate a part (percent) of a given whole. By converting 22% to its decimal equivalent we obtain

$$\frac{22}{100} \times \$185 = 0.22 \times \$185 = \$40.70$$

That is, 22% of \$185 is \$40.70.

- b.  $40\frac{1}{4}\%$  of \$140.25 becomes

$$40.25\% \times \$140.25 = 0.4025 \times \$140.25 = \$56.45$$

That is,  $40\frac{1}{4}\%$  of \$140.25 is \$56.45.

- c. In converting  $0.08\bar{3}\%$  to its decimal equivalent, we need to be careful to maintain the correct accuracy in calculations to have our answer accurate to the cent. Since 1% of \$5000 is \$50, then 0.1% of \$5000 is only \$5. Therefore, the answer will be a little less than \$5. For the answer to be accurate to the cent, we seek three-figure accuracy.

$$0.08\bar{3}\% \times \$5000 = 0.0008333 \times \$5000 = \$4.17$$

Therefore, \$4.17 is  $0.08\bar{3}\%$  of \$5000.

- d. Here the percentage is greater than 100% so the answer will be larger than the original value of \$50.

$$1.40 \times \$50 = \$70$$

Therefore, 140% of \$50 is \$70.

**TRAP****Decimal Equivalent of Percentages Smaller Than 1%**

When a percent is less than 1%, students sometimes forget to move the decimal two places to the left in order to obtain the decimal equivalent. For example, be clear on the distinction between 0.25% and 25%. The former is just  $\frac{1}{4}$  of 1%—the latter is 25 times 1%. Their decimal equivalents are 0.0025 and 0.25, respectively. In terms of equivalent fractions, 0.25% equals  $\frac{1}{400}$ , but 25% equals  $\frac{1}{4}$ .

**EXAMPLE 1.2J****A WORD PROBLEM REQUIRING THE USE OF PERCENTS**

A battery manufacturer encloses a rebate coupon for 15% off in a package of two AAA batteries retailing for \$6.29. What rebate does the coupon represent?

**SOLUTION**

In effect, the question is asking you to find 15% of the retail price.

$$\text{Rebate} = 0.15 \times \$6.29 = \$0.94$$

The manufacturer's 15% rebate on the batteries is equivalent to a cash rebate of \$0.94.





## CONCEPT QUESTIONS

1. If you want four-figure accuracy in your final result, what minimum number of figures must be retained in the values used in the calculations? **A) 4 B) 5 C) 6**
2. For a final result of approximately \$7000 to be accurate to the cent, what minimum number of figures must be retained in the values used in the calculations? **A) 6 B) 7 C) 8**
3. If a final result of the order of \$5 million is to be accurate to the nearest dollar, what minimum number of figures must be retained in the calculations? **A) 6 B) 7 C) 8**
4. If an interest rate (which could be greater than 10%) is to be calculated to the nearest 0.01%, what minimum number of digits must be retained in the numbers used to calculate the interest rate? **A) 3 B) 4 C) 5**

## EXERCISE 1.2

Answers to the odd-numbered problems are at the end of the book.

### CALCULATOR-FREE PROBLEMS

Express each of the following first as a decimal and then as a percent. Round your answers to two decimal places where rounding is needed.

- |  |                        |                   |                   |
|--|------------------------|-------------------|-------------------|
| a. $\frac{1}{10}$  | b. $\frac{2}{5}$       | c. $\frac{1}{4}$  | d. $\frac{3}{4}$  |
| e. $1\frac{1}{2}$  | f. $2\frac{1}{3}$      | g. $\frac{10}{5}$ | h. $5\frac{2}{3}$ |
| i. What is 25% of 80?  | j. What is 20% of 120? |                   |                   |
| k. Jeffrey puts 20% of every paycheque into a savings plan and 50% of the money in the savings plan is in a Tax-Free Savings Account (TFSA). If he earns \$1000 every paycheque, how much money is he putting into the TFSA? |                        |                   |                   |

### BASIC PROBLEMS

The following fractions and mixed numbers have terminating decimal equivalent forms. Express their decimal and percent equivalent forms to five-figure accuracy.

- |                     |                       |                       |                        |
|---------------------|-----------------------|-----------------------|------------------------|
| 1. $\frac{7}{8}$    | 2. $\frac{65}{104}$   | 3. $\frac{47}{20}$    | 4. $-\frac{9}{16}$     |
| 5. $-\frac{35}{25}$ | 6. $1\frac{7}{25}$    | 7. $\frac{25}{1000}$  | 8. $\frac{1000}{25}$   |
| 9. $2\frac{2}{100}$ | 10. $-1\frac{11}{32}$ | 11. $\frac{37.5}{50}$ | 12. $\frac{22.5}{-12}$ |

The following fractions and mixed numbers have repeating decimal equivalent forms. Express their decimal and percent equivalent forms in the repeating decimal notation. Show just the minimum number of decimal places needed to display the repeating digit or group of digits.

- |                    |                      |                      |                     |
|--------------------|----------------------|----------------------|---------------------|
| 13. $\frac{5}{6}$  | 14. $-\frac{8}{3}$   | 15. $7\frac{7}{9}$   | 16. $1\frac{1}{11}$ |
| 17. $\frac{10}{9}$ | 18. $-\frac{4}{900}$ | 19. $-\frac{7}{270}$ | 20. $\frac{37}{27}$ |

Round each of the following to four-figure accuracy.

- |               |              |               |               |
|---------------|--------------|---------------|---------------|
| 21. 11.3845   | 22. 9.6455   | 23. 0.5545454 | 24. 1000.49   |
| 25. 1.0023456 | 26. 0.030405 | 27. 40.09515  | 28. 0.0090909 |



**Convert each of the following fractions and mixed numbers to its decimal equivalent and percent equivalent values, rounded to five figures.**

29.  $\frac{1}{6}$

30.  $\frac{7}{6}$

31.  $\frac{1}{60}$

32.  $2\frac{5}{9}$

33.  $\frac{250}{365}$

34.  $\frac{15}{365}$

35.  $\frac{0.11}{12}$

36.  $\frac{0.095}{12}$

**Evaluate each of the following, accurate to the nearest cent or two decimal places.**

37.  $\$92\left(1 + 0.095 \times \frac{112}{365}\right)$

38.  $\$100\left(1 + 0.11 \times \frac{5}{12}\right)$

39.  $\$454.76\left(1 - 0.105 \times \frac{11}{12}\right)$

40.  $\frac{1}{(1 + 0.22)^3}$

41.  $\$1447\left(1 + \frac{0.18}{2}\right)^3\left(1 + \frac{0.21}{3}\right)^2$

42.  $\frac{\$790.84}{1 + 0.13 \times \frac{311}{365}}$

43.  $\frac{\$3490}{1 + 0.125 \times \frac{91}{365}}$

44.  $\frac{\$10,000}{1 - 0.10 \times \frac{182}{365}}$

45.  $\$650\left(1 + \frac{0.105}{2}\right)^2$

46.  $\$950.75\left(1 - \frac{0.095}{4}\right)^2$

47.  $\frac{\$15,400}{\left(1 + \frac{0.13}{12}\right)^6}$

48.  $\frac{\$550}{\left(1 + \frac{0.115}{2}\right)^4}$

49. What is  $33\frac{1}{3}\%$  of \$1527?

50. What is 2.75% of \$2.75?

51. What amount is 250% of \$25?

52. 0.025% of \$200 is what amount?

53. How much is  $\frac{1}{2}\%$  of \$30?

54. A local hydro company uses smart meters to measure hydro consumption. Off-peak rates in the winter months apply from 7 p.m. to 7 a.m., mid-peak hours apply from 11 a.m. to 5 p.m., and on-peak hours apply from 7 a.m. to 11 a.m. and again from 5 p.m. to 7 p.m. What percentage of the total daily hours is attributed to off-peak, mid-peak, and on-peak rates?

55. Mike and Laurie are building a home and have budgeted 5.5% of the \$458,000 construction cost for windows. If they purchase energy-efficient windows they can get a 12% rebate on the cost of the windows. What is the dollar value of the rebate on the windows?

56. Bethany and Irwin estimate their total cost for a vacation in Cuba to be \$14,775. If 53% of this cost is for flights and accommodations and 42% is for meals, how much money will they be able to spend on entertainment?

57. In the month of July, a convenience store had total sales of \$102,300 from its gas pumps and other in-store products. If the Harmonized Sales Tax (HST) is 13% of sales, how much HST was collected on the in-store products if these sales represent 36% of total sales?

58. In a basketball game, the Langara College Falcons scored  $54.\overline{54}\%$  of 33 shots from the 2-point zone,  $46.\overline{6}\%$  of 15 attempts from the 3-point distance, and 79.3% of 29 free throws (1 point each). How many points did the Falcons score?

### INTERMEDIATE PROBLEMS

**Evaluate each of the following, accurate to the nearest cent or two decimal places.**

59.  $\frac{\$6600\left(1 + 0.085 \times \frac{153}{365}\right)}{1 + 0.125 \times \frac{82}{365}}$

60.  $\frac{\$780\left(1 + \frac{0.0825}{2}\right)^5}{\left(1 + \frac{0.10}{12}\right)^8}$



$$61. \$1000 \left[ \frac{\left(1 + \frac{0.09}{12}\right)^7 - 1}{\frac{0.09}{12}} \right]$$

$$62. \frac{\$350}{\frac{0.0975}{12}} \left[ 1 - \frac{1}{\left(1 + \frac{0.0975}{12}\right)^5} \right]$$

$$63. \frac{\$9500}{\frac{\left(1 + \frac{0.075}{4}\right)^5 - 1}{\frac{0.075}{4}}}$$

$$64. \$45 \left[ \frac{1 - \frac{1}{\left(1 + \frac{0.0837}{2}\right)^4}}{\frac{0.0837}{2}} \right] + \frac{\$1000}{\left(1 + \frac{0.0837}{2}\right)^4}$$

65. The Calgary Flames hockey team announced that its season ticket sales represent 67.50% of the Scotiabank Saddledome's seating capacity of 19,289 seats. Rounded to the nearest 100, how many seats were not sold to season ticket holders?
66. The Royal Canadian Mint sells one-troy-ounce (31.16 g) platinum collector coins of 99.95% purity. How many milligrams of impurities are in a single coin?
67. Stan is a real estate salesperson. He receives 60% of the 4.8% commission that the real estate agency charges on sales. If his sales for the past year were \$5,225,000, what was the dollar value of his commission?
68. The maximum amount an individual can contribute to her Registered Retirement Savings Plan (RRSP) for a year is set from time to time by the regulations of the Income Tax Act. For the year 2015, the maximum contribution was the lesser of \$24,930 or 18% of the individual's "earned income" during 2014. What was the maximum RRSP amount that could be contributed in 2015 based on an income of \$128,500 in 2014?
69. The label on a single-serving can of soup states that it contains 35% of an adult's daily sodium intake. If the maximum recommended sodium intake is 2300 mg, how many grams of sodium need to be consumed daily through other foods?

### \*1.3 Payroll

**L05** An employee's **remuneration** may be based on an hourly wage, a salary, a piecework rate, or a rate of commission. In some cases, earnings are based on a combination of a commission with a wage or a salary. This section deals only with the calculation of *gross earnings*—the amount earned before any deductions.<sup>3</sup>

### Salaries

Where employment tends to be predictable and steady, an employee typically receives a salary quoted in terms of a biweekly, monthly, or annual amount. A monthly salary is usually paid on a monthly or semimonthly basis.<sup>4</sup> An annual salary may be paid at monthly, semimonthly, biweekly, or weekly intervals. For monthly and semimonthly pay periods, the gross earnings per pay are calculated by dividing the annual salary by 12 and 24, respectively.

For purposes of this section, we assume there are exactly 52 weeks in a year. Therefore, we can divide an annual salary by 26 or by 52 to obtain the gross earnings per biweekly or weekly pay period, respectively.<sup>5</sup>

<sup>3</sup> Employers are required by law to withhold income tax and the employee's share of Canada Pension Plan contributions and employment insurance premiums. By agreement with the employees or their union, an employer may also deduct and remit various insurance premiums, union dues, and pension plan contributions.

<sup>4</sup> Provincial employment standards usually provide that, if requested by the employees, monthly salaries be paid no less frequently than semimonthly. Some employers satisfy the requirement by providing a midmonth advance of approximately half the monthly take-home pay.

<sup>5</sup> A 365-day year contains 52 weeks *plus* one day; a leap year contains 52 weeks *plus* two days. As an approximate average, one year in six will have 53 Fridays, the customary payday for weekly and biweekly payrolls. Accordingly, approximately one year in six will have 53 weekly paydays, and one year in twelve will have 27 biweekly paydays. Employers must take this into account when converting annual salaries to weekly or biweekly rates of pay.



If a salaried employee has a specified length of regular pay period and he is eligible for an overtime rate of pay, the usual hourly overtime rate is

$$1.5 \times \frac{\text{Gross earnings for a regular pay period}}{\text{Hours in a regular pay period}}$$

Because of the factor 1.5, this overtime rate is usually referred to as “time and a half.”

**EXAMPLE 1.3A****CALCULATING BIWEEKLY AND WEEKLY EARNINGS AND HOURLY OVERTIME RATE FROM AN ANNUAL SALARY**

Benazir’s annual salary is \$45,000. Her regular workweek consists of four 10-hour workdays. She is eligible for overtime at “time and a half” on time worked in excess of 10 hours per day or 40 hours per week. Determine her gross earnings in a pay period if

- she is paid semimonthly.
- she is paid biweekly.
- she works 6 hours of overtime in a biweekly pay period.

**SOLUTION**

$$\text{a. Semimonthly gross pay} = \frac{\text{Annual salary}}{24} = \frac{\$45,000}{24} = \$1875.00$$

$$\text{b. Biweekly gross pay} = \frac{\text{Annual salary}}{26} = \frac{\$45,000}{26} = \$1730.77$$

$$\text{c. Regular hourly rate} = \frac{\text{Regular biweekly gross pay}}{\text{Regular hours in biweekly period}} = \frac{\$1730.77}{2 \times 4 \times 10} = \$21.635$$

$$\text{Overtime hourly rate} = \text{Overtime factor} \times \text{Regular hourly rate} = 1.5 \times \$21.635 = \$32.453$$

$$\text{Total gross earnings} = \text{Regular pay} + \text{Overtime pay} = \$1730.77 + 6(\$32.453) = \$1925.49$$

**Hourly Wages**

In jobs where the amount of work available is irregular or unpredictable, or where overtime is a common occurrence, employees are typically paid an hourly wage. Usually, a collective agreement between the employer and employees sets the number of hours per day (typically 7.5 or 8) and hours per week (typically 37.5 or 40) beyond which higher overtime rates apply. If no such agreement exists, federal or provincial employment standards laws apply. The minimum and most common overtime rate is 1.5 times the regular hourly rate (“time and a half”). Some unions have negotiated more favourable overtime rates (such as “double time”).

Each province recognizes certain holidays that, depending on the province, are called “statutory holidays,” “public holidays,” or “general holidays.” New Year’s Day, Good Friday, Canada Day, Labour Day, Thanksgiving Day, and Christmas Day are holidays common to all provinces. With a few exceptions, provincial employment standards require that employees receive their usual rate of pay for a statutory holiday *not* worked. If employees are required to work on a “stat” holiday, they must be paid an *additional premium* rate of 1.5 times their regular rate of pay.

You calculate the gross earnings for a pay period by adding overtime pay, “stat” holiday pay, and “stat” holiday premium pay to the regular pay. That is,

$$\begin{aligned} & \text{Regular hourly rate} \times \text{Regular hours} \\ & + \text{Overtime hourly rate} \times \text{Overtime hours} \\ & + \text{“Stat” holiday pay} \\ & + \text{“Stat” holiday premium hourly rate} \times \text{“Stat” holiday hours worked} \\ & = \text{Gross earnings} \end{aligned}$$



In calculations in this section, time worked on a “stat” holiday does *not* count toward the threshold for overtime hours in a week.<sup>6</sup>

Sometimes wages in production and manufacturing jobs are structured to create an incentive for higher productivity. A *piecework rate* is based on the unit of production, such as \$1 per garment sewn, or \$2 per typed page, or \$15 per tonne of output.

$$\left( \begin{array}{c} \text{Piecework} \\ \text{earnings} \end{array} \right) = \left( \begin{array}{c} \text{Number of} \\ \text{units produced} \end{array} \right) \times \left( \begin{array}{c} \text{Piecework} \\ \text{rate} \end{array} \right)$$

**EXAMPLE 1.3B      CALCULATING THE GROSS EARNINGS OF AN HOURLY PAID EMPLOYEE**

Steve is paid \$36.60 an hour for his work on an assembly line. The regular workweek is 37.5 hours (five 7.5-hour shifts). In the most recent biweekly pay period (midnight Friday to midnight of the second following Friday), he worked full shifts from Monday to Friday of both weeks. The first Monday of the pay period was a statutory holiday. In addition, he worked 6 hours on each Saturday. Overtime is paid at 1.5 times the regular rate and the statutory holiday time premium is 1.5 times the regular rate. What was Steve’s gross pay for the period?

**SOLUTION**

In addition to “stat” holiday pay for the first Monday, Steve will be paid a holiday premium rate of 1.5 times his regular rate for hours actually worked on that Monday. (These hours do not count toward the threshold of 37.5 hours per week for overtime eligibility.) Steve’s hourly rate for overtime is  $1.5 \times \$36.60 = \$54.90$ .

The given information is summarized in the following table.

Week 1			Week 2		
Day	Hours	Rate of pay	Day	Hours	Rate of pay
Sat	6	Regular	Sat	6	Regular
Sun	0		Sun	0	
Mon	7.5	Holiday premium	Mon	7.5	Regular
Tues	7.5	Regular	Tues	7.5	Regular
Wed	7.5	Regular	Wed	7.5	Regular
Thur	7.5	Regular	Thur	7.5	Regular
Fri	7.5	Regular	Fri	7.5	1.5 hours regular; 6 hours overtime

The components of Steve’s gross pay are:

Regular pay:	$[6 + 4(7.5) + 6 + 4(7.5) + 1.5]\$36.60 = \$2690.10$
Overtime pay:	$6(\$54.90) = \$ 329.40$
Holiday pay:	$7.5(\$36.60) = \$ 274.50$
Holiday premium pay:	$7.5(\$54.90) = \$ 411.75$
Total:	<u><u>\$3705.75</u></u>

Steve’s gross pay for the two-week period was \$3705.75.

<sup>6</sup> This is the basic standard established by Ontario’s Employment Standards Act. As with many employment standards set by provincial legislation and regulations, unions typically negotiate more favourable remuneration and working conditions in their collective agreements.



**EXAMPLE 1.3C**
**CALCULATING GROSS EARNINGS INCLUDING A PIECEWORK WAGE**

An orchardist pays apple pickers \$10 per hour plus \$8 per 100 kg of apples picked. If a worker picks, on average, 180 kg of apples per hour for a 40-hour workweek, what are the worker’s gross earnings for the week?

**SOLUTION**

$$\begin{aligned} \left( \begin{array}{c} \text{Gross} \\ \text{earnings} \end{array} \right) &= \left( \begin{array}{c} \text{Hourly} \\ \text{rate} \end{array} \times \begin{array}{c} \text{Number} \\ \text{of hours} \end{array} \right) + \left( \begin{array}{c} \text{Piecework} \\ \text{rate} \end{array} \times \begin{array}{c} \text{Number of} \\ \text{units} \end{array} \right) \\ &= (\$10.00 \times 40) + \$8.00 \left( \frac{180}{100} \times 40 \right) \\ &= \$400 + \$576 \\ &= \$976 \end{aligned}$$

The worker’s gross earnings for the week are \$976.

**Commissions**

For sales positions, it is standard practice to base at least a portion of the salesperson’s remuneration on sales volume. If earnings are calculated strictly as a percent of sales, the salesperson is working on *straight commission*. A *graduated commission* structure pays progressively higher commission rates at higher levels of sales. A salesperson who receives a basic salary and a commission on sales is working on a *salary plus commission* basis. In some arrangements, the commission is paid only on sales exceeding a minimum level called the *quota*.

**EXAMPLE 1.3D**
**CALCULATING GROSS EARNINGS BASED ON A SALARY PLUS COMMISSION**

James manages a men’s clothing store for a national chain. His monthly remuneration has three components: a \$2500 base salary, plus 2% of the amount by which the store’s total sales volume for the month exceeds \$40,000, plus 8% of the amount by which his personal sales exceed \$4000. Calculate his gross compensation for a month in which his sales totalled \$9900 and other staff had sales amounting to \$109,260.

**SOLUTION**

Base salary		\$2500.00
Commission on total store’s volume:	$0.02(\$109,260 + \$9900 - \$40,000)$	1583.20
Commission on personal sales:	$0.08(\$9900 - \$4000)$	472.00
Total compensation		\$4555.20

James’s gross earnings for the month are \$4555.20.

**EXAMPLE 1.3E**
**CALCULATING GROSS EARNINGS BASED ON A GRADUATED COMMISSION**

Tanya sells mutual funds for Pacific Financial Services Ltd. On mutual fund sales, Pacific Financial Services charges a “front-end load” or gross commission rate of 6%. Tanya is paid on a graduated commission structure. She receives 40% of the gross commission on the first \$100,000 worth of mutual funds she sells in a month, and 60% of the gross commission on all additional sales in the same month. What are her earnings for a month in which she sells \$180,000 worth of mutual funds?

**SOLUTION**

Commission on first \$100,000:	$0.40 \times 0.06 \times \$100,000$	\$2400
Commission on next \$80,000:	$0.60 \times 0.06 \times \$80,000$	2880
Total earnings		\$5280

Tanya’s earnings are \$5280 from her total mutual funds sales of \$180,000 in the month.



**EXERCISE 1.3**

Answers to the odd-numbered problems are at the end of the book.

**BASIC PROBLEMS**

*In this Exercise, assume there are exactly 52 weeks in a year.*

1. Aletta's annual salary of \$58,800 is paid weekly. She is paid at time and a half for any overtime beyond her regular workweek of 35 hours. What is her gross pay for a week in which she works 39 hours?
2. Lucille receives an annual salary of \$37,500 based on a 37.5-hour workweek. What are her gross earnings for a 2-week pay period in which she works 9 hours of overtime at 1.5 times her regular rate of pay?
3. Hasad is paid an annual salary of \$54,600 based on a 40-hour workweek. What is his gross pay for a biweekly pay period if he works 43 hours in the first week and 46.5 hours in the second week? Overtime is paid at time and a half.
4. Ross's compensation is to be changed from an hourly rate of \$31.50 for a 40-hour workweek to a salary paid semimonthly. What should he be paid semimonthly in order for his annual earnings to remain the same?
5. Allison's regular hourly rate of pay is \$17.70. She is paid time and a half for all work on weekends and for any time over 7.5 hours on weekdays. Calculate her gross earnings for a week in which she works 4.5, 0, 7.5, 8.5, 6, 6, and 9 hours on Saturday to Friday, respectively.
6. Sam is paid \$34.50 per hour as a power plant engineer. He is paid 1.5 times the regular rate for all time exceeding 8 hours in a day or 40 hours in a week. Statutory holidays worked are paid at double time (in addition to holiday pay). What are his gross earnings for a week in which he clocks 8, 9.5, 8, 8, 10, 0, and 8 hours on Saturday to Friday, respectively, where Monday is a statutory holiday?
7. Mary sews for a clothing manufacturer. She is paid \$7.50 per hour plus a piecework rate that depends on the type of garment in production. The current production run is men's shirts, for which she is paid \$3.00 for each unit exceeding her quota of 20 shirts in an 8-hour shift. What is her total pay for a regular workweek in which her output on successive days is 24, 26, 27, 28, and 30 shirts?
8. Herb packs fish in 500-g cans on a processing line. He is paid \$8.25 per hour plus \$0.18 per kilogram for production in excess of 500 kg in a 7.5-hour shift. How much will he earn per day if he packs 250 cans per hour?
9. Svetlana is an independent insurance broker placing various clients with any of several insurance companies. On homeowner insurance policies, each month she receives:
  - \$20 for each renewal of an existing policy;
  - \$35 for each policy placed with a new client; and
  - 5.5% of the annual premiums on all new and renewed policies written in the month.

In October, she placed 37 new client policies representing \$14,375 in annual premiums and 126 policy renewals representing \$47,880 in annual premiums. What amount did Svetlana earn in October?
10. Hillary sells cosmetics from her part-time home-based business. She receives a straight commission of 21% from her supplier. At the year-end, she also receives a 7% bonus on sales exceeding her annual quota of \$100,000. What will her gross annual earnings be for a year in which her average monthly sales are \$11,000?



11. Manfred is considering job offers of the same type of sales position from two retailers with similar product lines:
- Supreme Audio & Video is offering a base salary of \$2000 per month plus a 4% commission rate on sales.
  - Buy-Right Electronics will pay a base salary of \$1500 per month plus commission rates of 3% on the first \$25,000 of sales and 6% on additional sales in a month.
- Based on past experience in similar sales positions, Manfred is confident he can attain average monthly sales of \$55,000. At this level of sales, what would be his average gross earnings per month from each retailer?
12. A shoe salesman is paid the greater of \$600 per week or 11% of sales.
- What are his earnings for a week in which sales are \$5636?
  - At what volume of sales per week will he start to earn more from the commission-based compensation?
13. Tom sells mutual funds on a graduated commission structure. He receives 3.3% on the first \$50,000 of sales in a month, 4.4% on the next \$50,000, and 5.5% on all further sales. What are his gross earnings for a month in which he sells \$140,000 worth of mutual funds?

### INTERMEDIATE PROBLEMS

14. Sharon is a manufacturer's representative selling office furniture directly to businesses. She receives a monthly salary of \$2000 plus a 2.2% commission on sales exceeding her quota of \$150,000 per month.
- What are her earnings for a month in which she has \$227,000 in sales?
  - If her average monthly sales are \$235,000, what straight commission rate would generate the same average monthly earnings as her current basis of remuneration?
15. Julio is paid on a graduated commission scale of 5% on the first \$20,000 of sales in a month, 7.5% on the next \$20,000, and 10% on all additional sales.
- What is he paid for a month in which his sales are \$54,880?
  - What single commission rate on all sales would result in the same earnings for the month?
16. Karen works in a retail computer store. She receives a weekly base salary of \$300 plus a commission of 3% of sales exceeding her quota of \$20,000 per week. What are her sales for a week in which she earns \$630.38?
17. Jason's gross pay for August is \$3296.97 on sales totalling \$151,342. If his base salary is \$1500 per month, what is his rate of commission on sales exceeding his monthly quota of \$100,000?
18. Daniella's gross monthly earnings are based on commission rates of 4% on the first \$40,000 of sales, 5% on the next \$50,000, and 6% on all additional sales for the month. What is her sales total for a month in which she earns \$5350?
19. Trevor earns a base monthly salary of \$2000 plus a commission of 3% on sales exceeding his monthly quota of \$25,000. He receives a further 3% bonus on sales in excess of \$50,000. What must his sales be in order to gross \$4000 per month?



## 1.4 Simple and Weighted Averages

### Simple Average

**L06** The type of average initially encountered in basic mathematics is called the simple average. To calculate the simple average, simply (of course) add the values for all items and then divide by the number of items. That is,

$$\text{Simple average} = \frac{\text{Sum of the values}}{\text{Total number of items}} \quad (1-1)$$

This average should be used in cases where each item has the *same* importance or each value occurs the *same* number of times.

### Weighted Average

We will now consider a situation requiring a different approach to averaging. Suppose you operate a seasonal business that employs 10 people during the peak period of July, August, and September. Only 2 employees are retained during the rest of the year. Is the average number of people employed during the year  $\frac{10+2}{2} = 6$ ? No—a simple average of “10” and “2” is not appropriate because these two employment levels lasted for different lengths of time. The value “2” should influence the average more than the value “10.” More precisely, each employment value should influence the average in proportion to the length of time at the level of employment. Mathematicians have a rather odd way of expressing this sort of idea. In this case, they say: “Each employment level should be *weighted* by the time period for which it lasted.” Consequently, we assign a *weighting factor* of 3 months to the value “10” and a weighting factor of 9 months to the value “2.” Then the *weighted average* number of employees during the year is calculated as follows:

$$\frac{(3 \times 10) + (9 \times 2)}{3 + 9} = 4.0$$

In the numerator, each of the two employment values (10 and 2) is multiplied by its weighting factor (this counts 3 months when there were 10 employees and 9 months when there were 2 employees). The two products are then added. This sum is then divided by the sum of the weighting factors, which represents the employment levels for 12 months.

In general, *a weighted average should be calculated when the values being averaged have differing relative importance, or when some values occur more often than others.*

$$\text{Weighted average} = \frac{\text{Sum of (Weighting factor} \times \text{Value)}}{\text{Sum of weighting factors}} \quad (1-2)$$

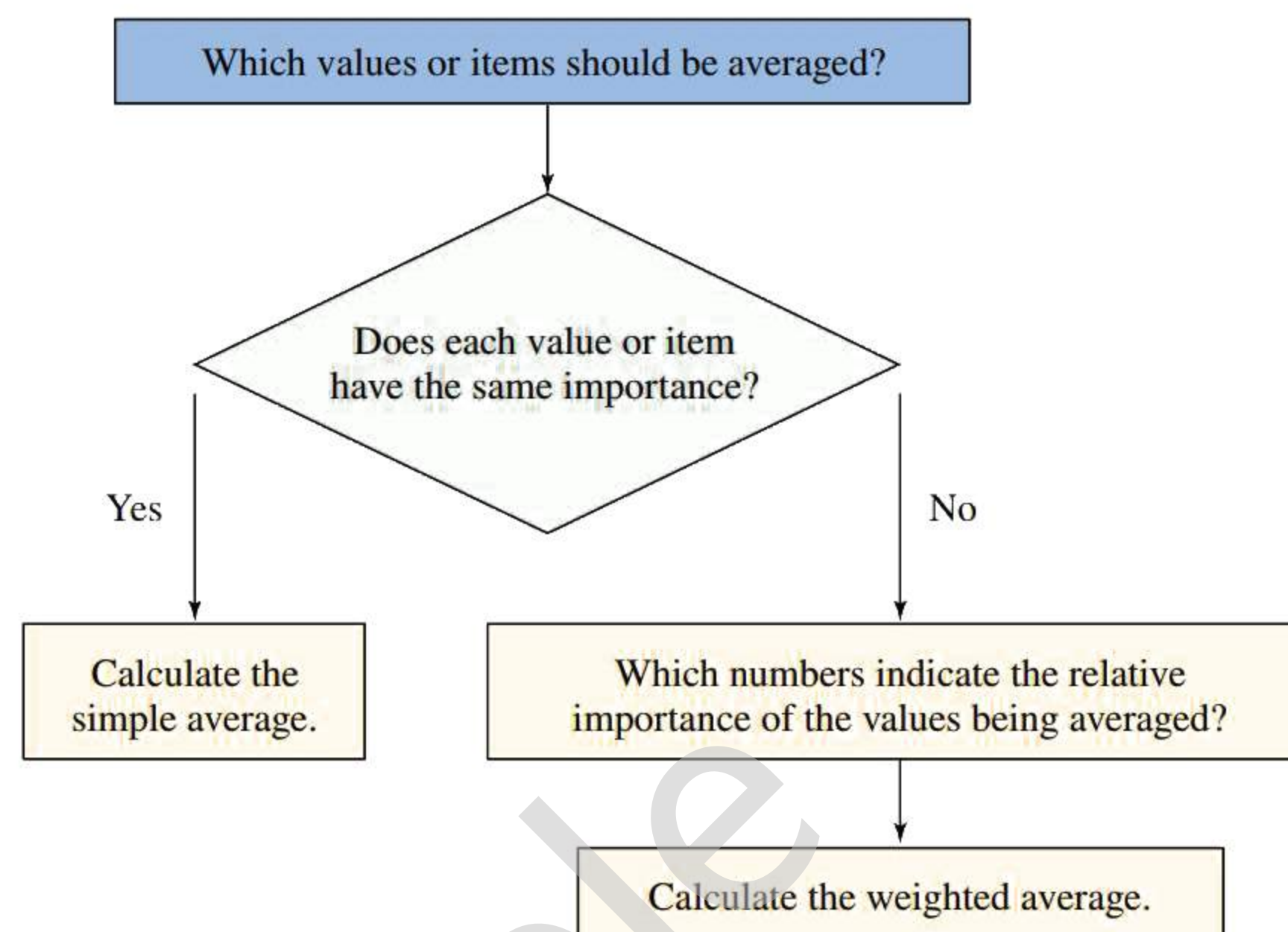
Formula (1-2)<sup>7</sup> implies three steps for calculating a weighted average:

1. First multiply each of the “values” by its “weighting factor.” The weighting factors represent the relative importance of each value, or the number of times each value occurs.
2. Add all of the products calculated in Step 1.
3. Finally, divide the Step 2 result by the sum of the “weighting factors.”

Weighted averages are frequently encountered in business. For example, the Toronto Stock Exchange’s S&P/TSX Composite Index is based on a weighted average price of the shares of over 200 companies. Accountants sometimes use the weighted average price paid for goods to determine the overall value of a firm’s inventory. Several more examples are presented in this section’s Example problems and Exercise problems. When calculating weighted averages, you need to make careful decisions about which values should be averaged and which numbers should be used for the weighting factors. The flow chart in **Figure 1.1** suggests questions you should ask yourself before the “number crunching” begins.

<sup>7</sup> Note that if each of the “values” has the same importance, then each weighting factor is 1. The “weighted average” Formula (1-2) then reduces to the “simple average” Formula (1-1).



**FIGURE 1.1** Approach for Problems on Averages**EXAMPLE 1.4A** CALCULATION OF SIMPLE AND WEIGHTED AVERAGES

Northern Transport has 86 drivers each earning \$23.85 per hour, 14 clerical staff members each earning \$18.50 per hour, and 8 mechanics each earning \$35.50 per hour.

- What is the simple average of the three hourly rates?
- Calculate the weighted average hourly rate earned by the three categories of employees.

**SOLUTION**

a. Simple average =  $\frac{\$23.85 + \$18.50 + \$35.50}{3} = \$25.95$

- b. Each hourly rate should be assigned a weighting factor reflecting the relative importance of that rate. The greater the number of employees receiving a particular wage rate, the more importance should be given to that rate. It is natural, then, to use the number of employees receiving each hourly rate as the weighting factors.

$$\begin{aligned}
 \text{Weighted average} &= \frac{(86 \times \$23.85) + (14 \times \$18.50) + (8 \times \$35.50)}{86 + 14 + 8} \\
 &= \frac{\$2051.10 + \$259.00 + \$284.00}{108} \\
 &= \$24.02
 \end{aligned}$$

The weighted average is less than the simple average because a high proportion of the employees earn the two lower hourly rates.

**EXAMPLE 1.4B** CALCULATING THE WEIGHTED AVERAGE RATE OF RETURN FOR AN INVESTMENT PORTFOLIO

One year ago, Mrs. Boyd divided her savings among four mutual funds as follows: 20% was invested in a bond fund, 15% in a money market fund, 40% in a Canadian equity fund, and 25% in a global equity fund. During the past year, the rates of return on the individual mutual funds were 10%, 4%, -2%, and 15%, respectively. What was the overall rate of return on her portfolio?



**SOLUTION**

A simple average of the four rates of return is not the appropriate calculation because Mrs. Boyd invested different amounts of money in each mutual fund. The  $-2\%$  return on the Canadian equity fund should have twice the influence of the  $10\%$  return on the bond fund because she invested twice as much money in the equity fund as in the bond fund. Therefore, we should choose weighting factors that reflect the relative amount of money invested in each mutual fund.

Mutual fund	Rate of return (Value)	Fraction of money invested (Weighting factor)	(Weighting factor) $\times$ (Value)
Bond fund	10%	0.20	2.0%
Money market fund	4%	0.15	0.6%
Canadian equity fund	$-2\%$	0.40	$-0.8\%$
Global equity fund	15%	0.25	3.75%
Total:		1.00	5.55%

The weighted average rate of return on the portfolio was  $\frac{5.55\%}{1.0} = 5.55\%$ .

Mrs. Boyd's portfolio increased in value by 5.55%.



**TIP**

**Determining the Weight Factor**

In some questions it is not always clear which values are to be used as the weight factor, particularly when all values are measured in the same units as in **Example 1.4B**. Every word problem will ask a question indicating which of the values is to be averaged—the other value is then used as the weight factor.

**EXAMPLE 1.4C**

**CALCULATING THE WEIGHTED AVERAGE OF A VARYING INVESTMENT IN A BUSINESS**

As of January 1, Alan had already invested \$63,000 in his business. On February 1 he invested another \$5000. Alan withdrew \$12,000 on June 1 and injected \$3000 on November 1. What was his average cumulative investment in the business during the year? (Assume that all months have the same length, or weighting.)

**SOLUTION**

A common error made in this type of problem is to attempt, in some way, to average the amounts that are contributed to or withdrawn from the business. We should instead average the cumulative balance of the invested funds. The amounts contributed and withdrawn from time to time are used only to revise the cumulative investment. A weighted average should be calculated since the various amounts are invested for *differing* lengths of time. Each cumulative investment should be weighted by the number of months for which it lasted.

Period	Cumulative investment	Number of months
Jan. 1–Jan. 31	\$63,000	1
Feb. 1–May 31	$\$63,000 + \$5000 = \$68,000$	4
June 1–Oct. 31	$\$68,000 - \$12,000 = \$56,000$	5
Nov. 1–Dec. 31	$\$56,000 + \$3000 = \$59,000$	2

$$\begin{aligned} \text{Average investment} &= \frac{(1 \times \$63,000) + (4 \times \$68,000) + (5 \times \$56,000) + (2 \times \$59,000)}{12} \\ &= \$61,083.33 \end{aligned}$$

Alan's average investment in the business was \$61,083.33.



**EXAMPLE 1.4D** CALCULATING A (WEIGHTED) GRADE POINT AVERAGE

Most colleges compute a grade point average (GPA) as the measure of a student's overall academic achievement. To compute the GPA, each letter grade is first converted to a grade point value. Each course's grade point value is then weighted by the number of credits the course carries.

The first table gives City College's scale for converting letter grades to grade point values. The second table presents Louise's courses and grades. Calculate her GPA.

Letter grade	Grade point value	Course	Credits	Grade
A	4.0	English 100	3	B+
A–	3.7	Math 100	4	B
B+	3.3	Business 100	2	A
B	3.0	Economics 120	3	B–
B–	2.7	Accounting 100	4	C+
C+	2.3	Marketing 140	2	A–
C	2.0	Computing 110	3	C
C–	1.7	Total	21	
D	1.0			

**SOLUTION**

The values to be averaged are the grade point scores Louise has achieved on her seven courses. However, these values are not given in a ready-to-use list. A new table should be constructed, showing the grade points earned on each course. (See the first three columns in the following table.)

A simple average of the grade point values is not appropriate because some courses carry more credits than others. A 4-credit course should count twice as much in the average as a 2-credit course. Therefore, each course's grade point score should be weighted by the number of credits that course carries (Column 4). The first step in the calculation of the weighted average is to multiply each grade point value (Column 3) by its weighting factor (Column 4). In the second step, the products in Column 5 are added.

Course	Grade	Grade points	Credits	Grade points $\times$ Credits
English 100	B+	3.3	3	9.9
Math 100	B	3.0	4	12.0
Business 100	A	4.0	2	8.0
Economics 120	B–	2.7	3	8.1
Accounting 100	C+	2.3	4	9.2
Marketing 140	A–	3.7	2	7.4
Computing 110	C	2.0	3	6.0
Total:			21	60.6

The third and last step is to divide the total in Column 5 by the total in Column 4. Hence,

$$\text{GPA} = \frac{60.6}{21} = 2.89$$

Louise's grade point average is 2.89.





## CONCEPT QUESTIONS

1. In what circumstance should you calculate a weighted average instead of a simple average?
2. In what circumstance is the weighted average equal to the simple average?
3. How must you allocate your money among a number of investments so that your portfolio's overall rate of return will be the same as the simple average of the rates of return on individual investments?

## EXERCISE 1.4

Answers to the odd-numbered problems are at the end of the book.

### BASIC PROBLEMS

1. A survey of 254 randomly chosen residences in a city revealed that 4 had four television sets, 22 had three TV sets, 83 had two TV sets, 140 had one TV set, and 5 had no TV set at all. Based on the survey, what would you estimate to be the average number of TV sets per household?
2. An investor accumulated 1800 shares of Microtel Corporation over a period of several months. She bought 1000 shares at \$15.63, 500 shares at \$19.00, and 300 shares at \$21.75. What was her average cost per share? (*Note: Investors who purchase shares in the same company or the same mutual fund at more than one price must eventually do this calculation. Tax rules require that the capital gain or loss on the sale of any of the shares be calculated using the weighted average price paid for all of the shares, rather than the particular price paid for the shares actually sold.*)
3. A hockey goalie's "goals against average" (GAA) is the average number of goals scored against him per (complete) game. In his first 20 games in goal, O. U. Sieve had one shutout, two 1-goal games, three 2-goal games, four 3-goal games, seven 4-goal games, two 6-goal games, and one 10-goal disaster. Calculate his GAA.
4. Serge's graduated commission scale pays him 3% on his first \$30,000 in sales, 4% on the next \$20,000, and 6% on all additional sales in a month. What is his average commission rate on sales for a month totalling:
  - a. \$60,000?
  - b. \$100,000?
5. A baseball player's slugging percentage is the average number of hits a batter gets in all the "at bats" they have in their games played. This statistic differs from their batting average in that it is weighted based on the number of bases run for a single hit. Babe Ruth had 8,399 "at bats" in his career where he had 714 home runs (4-base hits), 136 triples (3-base hits), 506 doubles (2-base hits), and 2873 singles (1-base hits). What was his slugging percentage?
6. Brianne spent the summer touring Canadian cities. She spent 4 nights in a hotel in Montreal at \$158 a night, 2 nights in a Toronto hotel at \$199 a night, 1 night in Niagara Falls for \$239, 5 nights in Calgary at \$130 a night, and 2 nights in Kamloops for \$118 a night. What was the average cost of her hotel rooms for the entire trip?
7. The RBC Royal Bank offers an "add-on option" on fixed-rate mortgages. The option allows the customer to borrow additional funds partway through the term of the mortgage. The interest rate charged on the combined mortgage debt is the weighted average of the old rate on the former balance and the current competitive rate on new mortgage financing. Suppose Herschel and Julie had a mortgage balance of \$37,500 at 8%, when they borrowed another \$20,000 at 7%. What interest rate are they charged by the RBC Royal Bank on the new consolidated balance?



8. Margot’s grades and course credits in her first semester at college are listed below.

Grade	C+	B–	B+	C–	B	C
Credits	5	3	4	2	3	4

Using the table in **Example 1.4D** for converting letter grades to grade point values, calculate Margot’s grade point average for the semester.

9. The distribution of scores obtained by 30 students on a quiz marked out of 10 is listed below.

Score	10	9	8	7	6	5	4	3	2	1
Number of students	2	6	9	7	3	2	0	1	0	0

What was the average score on the test?

10. Alihan’s transcript shows the following academic record for four semesters of part-time college studies. Calculate his cumulative GPA at the end of his fourth semester.

Semester	Credits	GPA
1	6	3.5
2	9	3.0
3	12	2.75
4	7.5	3.2

11. The “age” of an account receivable is the length of time that it has been outstanding. At the end of October, a firm has \$12,570 in receivables that are 30 days old, \$6850 in receivables that are 60 days old, and \$1325 in receivables that are 90 days old. What is the average age of its accounts receivable at the end of October?
12. One year ago, Sook-Yin allocated the funds in her portfolio among five securities in the proportions listed below. The rate of return on each security for the year is given in the third column of the table.

Security	Proportion invested (%)	Rate of return for the year (%)
Company A shares	15	14
Province B bonds	20	10
Company C shares	10	–13
Units in Fund D	35	12
Company E shares	20	27

Calculate the rate of return for the entire portfolio.

13. One of the methods permitted by Generally Accepted Accounting Principles (GAAP) for reporting the value of a firm’s inventory is *weighted average inventory pricing*. The Boswell Corporation began its fiscal year with an inventory of 156 units valued at \$10.55 per unit. During the year it made the purchases listed in the following table.

Date	Units purchased	Unit cost (\$)
February 10	300	10.86
June 3	1000	10.47
August 23	500	10.97

At the end of the year, 239 units remained in inventory. Determine:

- a. The weighted average cost of the units purchased during the year.
  - b. The weighted average cost of the beginning inventory and all units purchased during the year.
  - c. The value of the ending inventory based on the weighted average cost calculated in part (b).
14. Suppose a group of consumers spend 30% of their disposable income on food, 20% on clothing, and 50% on rent. If over the course of a year the price of food rises 10%, the price of clothing drops 5%, and rent rises 15%, what is the average price increase experienced by these consumers?



INTERMEDIATE PROBLEMS

15. A restaurant owner sets her menu prices at a predetermined percentage of her input costs for food, ingredients, and beverages. The second column of the following table shows the prices as a percentage of these costs for various menu categories. The third column shows the breakdown of the restaurant's overall revenue from the four menu categories.
- a. On average, what are menu prices as a percentage of the basic input costs?
  - b. Overall, what are the input costs as a percentage of revenue?

Menu category	Menu price as a percentage of costs	Percentage of sales revenue
Appetizers	300%	10%
Entrees	200%	50%
Desserts	225%	15%
Beverages	250%	25%

16. The balance on Nucorp's revolving loan began the month at \$35,000. On the eighth of the month another \$10,000 was borrowed. Nucorp was able to repay \$20,000 on the 25th of the 31-day month. What was the average balance on the loan during the month? (Use each day's closing balance as the loan balance for the day.)
17. A seasonal manufacturing operation began the calendar year with 14 employees. During the year, employees were hired or laid off on various dates as shown in the following table.

Date	Employee changes
April 1	7 hired
May 1	8 hired
June 1	11 hired
September 1	6 laid off
October 1	14 laid off

What was the average number of employees on the payroll during the calendar year? (Assume that each month has the same length.)

18. Marcel must temporarily invest extra money in his retail business every fall to purchase additional inventory for the Christmas season. On September 1, he already had a total of \$57,000 invested in his business. Subsequently, he invested or withdrew cash on various dates as shown.

Date	Additional investment or withdrawal
October 1	\$15,000 investment
November 1	\$27,000 investment
February 1	\$23,000 withdrawal
March 1	\$13,000 withdrawal
May 1	\$6000 withdrawal

What was the average cumulative investment in the business during the period from September 1 to August 31? (Assume that each month has the same length.)

19. When a company calculates its earnings per common share for its financial statements, it uses the weighted average number of common shares outstanding during the year. Enertec Corp. began its fiscal year (January 1 to December 31) with 5 million common shares outstanding. Additional common shares were issued during the year as shown in the following table.

Date	Event	Additional shares issued
March 1	New public offering	1,000,000
June 1	Employees and officers exercise stock options	500,000
November 1	Convertible bonds exchanged for shares	750,000

What was average number of common shares outstanding during the year? (Assume that each month has the same length.)



20. Lien, the proprietor of a grocery store, prepares her Deluxe Nut Combo Mix by mixing various ingredients she buys in bulk. The second column of the following table shows the amount of each ingredient Lien uses in making a batch of the combo mix. To set the retail price of the mix at 150% of her cost, Lien must determine her cost based on the average wholesale cost of the ingredients, which is shown in the third column.

Ingredient	Amount	Cost per kg
Peanuts	5 kg	\$2.95
Cashews	2 kg	\$9.50
Almonds	1 kg	\$11.50
Sunflower seeds	500 g	\$2.75
Raisins	400 g	\$3.60
Smarties	300 g	\$6.40

- What is Lien's average cost per 100 g of her Deluxe Nut Combo Mix?
- What is her retail price per 100 g?

## \*1.5 Taxes

**L07** A **tax rate** is the percentage of a price or taxable amount that must be paid in tax. The dollar amount of the tax payable is

$$\text{Tax payable} = \text{Tax rate} \times \text{Taxable amount}$$

This word equation is an application of finding the percent of a number as shown in [Section 1.2](#).

### Goods and Services Tax (GST); Harmonized Sales Tax (HST)

The Goods and Services Tax (GST) is a federal sales tax applied to the vast majority of goods and services. The tax is initially collected by the seller of a good or service from the purchaser. Consequently, a business *collects* the GST on the prices of goods and services it sells to customers, and it *pays* the GST on the prices of goods and services it buys from suppliers. As of January 1, 2008, the GST rate is 5%.

A business remits GST payments monthly or quarterly to the Canada Revenue Agency (CRA). The GST remittance is the amount by which the tax collected from sales exceeds the tax paid on purchases during the reporting period. In sales-tax jargon, the amount by which sales (the business's "outputs") exceeds purchases (the business's "inputs") is called the "value added." Consequently, the GST is a type of sales tax often described as a "value-added tax" because the GST rate is, in effect, applied to the

$$\text{Value added} = \text{Value of "outputs"} - \text{Value of "inputs"}$$

Then

$$\text{GST remittance} = 0.05 \times \text{Value added}$$

If, in a reporting period, the GST collected from sales happens to be less than the GST paid on purchases, the CRA will refund the difference to the business.

Five provinces (PEI, New Brunswick, Newfoundland and Labrador, Nova Scotia, and Ontario) have agreed with the federal government to blend their respective provincial sales tax with the GST in a single Harmonized Sales Tax (HST). The HST rates are given in [Table 1.1](#). The HST is administered by the federal government through the CRA. Like the GST, the HST is a value-added tax.



**TABLE 1.1 GST, HST, and PST Rates (as of January 1, 2016)**

Province	GST rate (%)	HST rate (%)	PST rate (%)
Alberta	5	—	—
British Columbia	5	—	7
Manitoba	5	—	8
New Brunswick	—	13	—
Newfoundland & Labrador	—	13	—
Nova Scotia	—	15	—
Ontario	—	13	—
Prince Edward Island	—	14	—
Quebec	5	—	9.975
Saskatchewan	5	—	5

**EXAMPLE 1.5A****CALCULATION OF THE GST OR THE HST PAYABLE BY A BUSINESS**

Ace Appliance Repair files GST returns quarterly. During the first quarter of the year, Ace billed its customers \$17,650 for labour, \$4960 for parts, and then added the GST. In the same period, Ace paid \$3250 to suppliers for parts, \$1800 for rent, \$673 for utilities, \$594 for truck repairs, plus the GST on these goods and services.

- What GST must be remitted by Ace (or refunded by the CRA) for the first quarter?
- Repeat part (a) for the case where Ace also bought a new truck for \$36,000 on the last day of the quarter.
- Repeat part (a) for the case where Ace pays HST (at the rate of 13%) instead of GST.

**SOLUTION**

- $\text{GST collected} = 0.05(\$17,650 + \$4960) = \$1130.50$   
 $\text{GST paid} = 0.05(\$3250 + \$1800 + \$673 + \$594) = \$315.85$   
 $\text{GST remittance payable} = \$1130.50 - \$315.85 = \$814.65$
- $\text{GST paid on the truck purchase} = 0.05 \times \$36,000 = \$1800.00$

This GST credit exceeds the net GST calculated in part (a). Hence, Ace will qualify for a refund.

$$\begin{aligned}
 \text{GST refund receivable} &= \$1800.00 - \text{Result in part (a)} \\
 &= \$1800.00 - \$814.65 \\
 &= \$985.35
 \end{aligned}$$

- $\text{HST collected} = 0.13(\$17,650 + \$4960) = \$2939.30$   
 $\text{HST paid} = 0.13(\$3250 + \$1800 + \$673 + \$594) = \$821.21$   
 $\text{HST remittance payable} = \$2939.30 - \$821.21 = \$2118.09$

**Provincial Sales Tax (PST)**

Manitoba, British Columbia, Quebec, and Saskatchewan charge a sales tax at the *retail* level. This tax typically applies to a somewhat narrower range of goods and services than the GST and HST. The provincial sales tax (PST) rates for these provinces are shown in **Table 1.1**. The tax rate is applied to the retail price. That is,

$$\text{PST} = \text{Sales tax rate} \times \text{Retail price}$$



**EXAMPLE 1.5B**
**CALCULATING THE PST, GST, AND HST**

- Calculate the total sales taxes on a \$100 item in
- a. Manitoba
  - b. Quebec
  - c. Prince Edward Island

**SOLUTION**

- a.  $\text{GST} + \text{PST} = (0.05 + 0.08) \times \$100 = \$13.00$
- b.  $\text{GST} + \text{PST} = (0.05 + 0.09975) \times \$100 = \$14.98$
- c.  $\text{HST} = 0.14 \times \$100 = \$14.00$

**Property Tax**

Real estate property tax is not a sales tax, but rather an annual tax paid by the owners of real estate. Property taxes are paid to municipal and regional governments to cover costs of municipal services, public schools, policing, etc. Tax rates are set by municipal and regional governments, and by other agencies (such as school boards) authorized by the provincial government to levy property taxes. The taxable value (usually called the *assessed value*) of each property is set by a provincial agency. Assessed values are periodically revised to keep them in line with market values.

Property tax rates in most provinces east of Manitoba are quoted in terms of dollars of tax per \$100 of assessed value. For example, if the rate for a municipality is \$1.55 per \$100 of assessed value, the property tax on a home assessed at \$200,000 will be

$$\frac{\$1.55}{\$100} \times \$200,000 = \$3100$$

A tax rate of \$1.55 per \$100 of assessed value is equal to 1.55% of assessed value. Most municipalities in Ontario now specify property tax rates simply as a percentage of the assessed value.

Provinces west of Ontario quote property tax rates as mill rates. A **mill rate** indicates the amount of tax per \$1000 of assessed value. For example, a mill rate of 13.732 means a tax of \$13.732 per \$1000 of assessed value. The percent equivalent of this mill rate is 1.3732%. If you choose to work with a mill rate rather than its percent equivalent, then

$$\text{Property tax} = \frac{\text{Mill rate}}{1000} \times \text{Assessed value of the property}$$

**EXAMPLE 1.5C**
**CALCULATING THE PROPERTY TAX ON A RESIDENTIAL PROPERTY**

A homeowner’s tax notice lists the following mill rates for various local services and capital developments. If the assessed value of the property is \$164,500, calculate each tax levy and the current year’s total property taxes.

Tax rate	Mill rate
Schools	6.7496
General city	7.8137
Water	0.8023
Sewer and sanitation	0.7468



**SOLUTION**

$$\begin{aligned}\text{School tax levy} &= \frac{\text{School mill rate}}{1000} \times \text{Assessed value} \\ &= \frac{6.7496}{1000} \times \$164,500 = \$1110.31\end{aligned}$$

Similarly,

$$\text{General city levy} = \frac{7.8137}{1000} \times \$164,500 = \$1285.35$$

$$\text{Water levy} = \frac{0.8023}{1000} \times \$164,500 = \$131.98$$

$$\text{Sewer levy} = \frac{0.7468}{1000} \times \$164,500 = \$122.85$$

$$\begin{aligned}\text{Total property taxes} &= \$1110.31 + \$1285.35 + \$131.98 + \$122.85 \\ &= \$2650.49\end{aligned}$$

**EXAMPLE 1.5D CALCULATING A PROPERTY TAX RATE**

The town council of Concord has approved a new capital levy component of the property tax to pay for a new recreation complex. The levy must raise \$400,000 in each of the next 10 years.

- If the total assessed value of properties within Concord's jurisdiction is \$738 million, what tax rate (to five-figure accuracy) must be set for the capital levy? Determine the rate both as a mill rate and as dollars per \$100.
- As a result of the capital levy, what additional tax will be paid each year by the owner of a property assessed at \$200,000?

**SOLUTION**

- Let us first calculate the tax rate as a percentage.

$$\begin{aligned}\text{Capital levy tax rate} &= \frac{\text{Required total tax}}{\text{Total assessed value}} \times 100\% \\ &= \frac{\$400,000}{\$738,000,000} \times 100\% \\ &= 0.054201\%\end{aligned}$$

Therefore, the tax rate for the extra capital levy is \$0.054201 per \$100 of assessed value. This corresponds to \$0.54201 per \$1000 of assessed value. Hence, the mill rate is 0.54201.

- With the tax levy quoted as \$0.054201 per \$100 of assessed value,

$$\begin{aligned}\text{Additional tax} &= \frac{\$0.054201}{\$100} \times \$200,000 \\ &= \$108.40\end{aligned}$$

With the tax levy quoted as 0.54201 mill,

$$\begin{aligned}\text{Additional tax} &= \frac{0.54201}{\$1000} \times \$200,000 \\ &= \$108.40\end{aligned}$$



**EXERCISE 1.5**

Answers to the odd-numbered problems are at the end of the book.

**BASIC PROBLEMS**

1. Johnston Distributing Inc. files quarterly GST returns. The purchases on which it paid the GST and the sales on which it collected the GST for the last four quarters were as follows:

Quarter	Purchases (\$)	Sales (\$)
1	596,476	751,841
2	967,679	627,374
3	823,268	1,231,916
4	829,804	994,622

Calculate the GST remittance or refund due for each quarter.

2. Sawchuk's Home and Garden Centre in Toronto files monthly HST returns. The purchases on which it paid the HST and the sales on which it collected the HST for the last four months were as follows:

Month	Purchases (\$)	Sales (\$)
March	135,650	57,890
April	213,425	205,170
May	176,730	313,245
June	153,715	268,590

Based on an HST rate of 13%, calculate the HST remittance or refund due for each month.

3. Calculate the total amount, including both GST and PST, that an individual will pay for a car priced at \$39,500 in
- Alberta.
  - Saskatchewan.
  - Quebec.
4. How much more will a consumer pay for an item listed at \$1000 (pretax) in Prince Edward Island than in Manitoba?
5. Angie's Flower Shop charges 13% Harmonized Sales Tax (HST) on all purchases.
- How much HST will she report for a plant priced at \$39.45?
  - As of February 4, 2013, if a consumer pays cash and cannot give the exact change, the total amount of the transaction must be rounded up or down to the nearest five cents. How much change will be given if the above purchase is paid for with a \$50 bill?
6. To attract shoppers, retailers occasionally advertise something like "Pay no HST!" Needless to say, neither the federal nor the provincial government is willing to forgo its sales tax. In this situation, the retailer must calculate and remit the HST as though the "ticket" price already includes these sales taxes. How much HST must a retailer in New Brunswick report on a \$495 item that he sells on a "Pay no HST" basis? (*Hint: What percentage is the HST of a HST-inclusive price?*)
7. What are the taxes on a property assessed at \$227,000 if the mill rate is 16.8629?
8.
  - Express a property tax rate increase of 0.1 mill in terms of dollars per \$100 of assessed value.
  - If the mill rate increases by 0.1 mill, what is the dollar increase in property taxes on a \$200,000 home?
9. The assessment on a farm consists of \$143,000 for the house and \$467,000 for the land and buildings. A mill rate of 15.0294 applies to residences, and a rate of 4.6423 applies to agricultural land and buildings. What are the total property taxes payable on the farm?



10. The assessed value on a property increased from \$285,000 last year to \$298,000 in the current year. Last year's property tax rate was \$1.56324 per \$100 of assessed value.
- What will be the change in the property tax from last year if the new tax rate is set at \$1.52193 per \$100?
  - What would the new tax rate have to be for the dollar amount of the property taxes to be unchanged?

### INTERMEDIATE PROBLEMS

11. The school board in a municipality will require an extra \$2,430,000 for its operating budget next year. The current mill rate for the school tax component of property taxes is 7.1253.
- If the total of the assessed values of properties in the municipality remains at the current figure of \$6.78 billion, what value must next year's school mill rate be set at?
  - If the total of all assessed values rises by 5% over this year's aggregate assessment, what value must next year's school mill rate be set at?
12. The total assessed value of property in Brockton has risen by \$97 million from last year's figure of \$1.563 billion. The property tax rate last year for city services was \$0.94181 per \$100 of assessed value. If the city's budget has increased by \$750,000, what tax rate should it set for the current year?

### KEY TERMS

Complex fraction  
Denominator  
Equivalent fractions  
Improper fraction

Mill rate  
Mixed number  
Numerator  
Proper fraction

Remuneration  
Tax rate

### SUMMARY OF NOTATION AND KEY FORMULAS

**FORMULA (1-1)**  $\text{Simple average} = \frac{\text{Sum of the values}}{\text{Total number of items}}$

**FORMULA (1-2)**  $\text{Weighted average} = \frac{\text{Sum of (Weighting factor} \times \text{Value)}}{\text{Sum of weighting factors}}$

### REVIEW PROBLEMS

Answers to the odd-numbered review problems are at the end of the book.

### BASIC PROBLEMS

1. **L01** Evaluate each of the following:

- $(2^3 - 3)^2 - 20 \div (2 + 2^3)$
- $4(2 \times 3^2 - 2^3)^2 \div (10 - 4 \times 5)$
- $\$213.85(1 - 0.095 \times \frac{5}{12})$
- $\frac{\$2315}{1 + 0.0825 \times \frac{77}{365}}$
- $\$325.75 \left(1 + \frac{0.105}{4}\right)^2$
- $\frac{\$710}{\left(1 + \frac{0.0925}{2}\right)^3}$
- $\$885.75 \left(1 + 0.0775 \times \frac{231}{365}\right) - \frac{\$476.50}{1 + 0.0775 \times \frac{49}{365}}$
- $\$859 \left(1 + \frac{0.0825}{12}\right)^3 + \frac{\$682}{\left(1 + \frac{0.0825}{12}\right)^2}$



2. **L01** Evaluate each of the following:
  - a.  $96 - (6 - 4^2) \times 7 - 2$
  - b.  $81 \div (5^2 - 16) - 4(2^3 - 13)$
  - c.  $\frac{\$827.69}{1 + 0.125 \times \frac{273}{365}} + \$531.49 \left( 1 + 0.125 \times \frac{41}{365} \right)$
  - d.  $\$550.45 \left( 1 + 0.0875 \times \frac{195}{365} \right) - \frac{\$376.29}{1 + 0.0875 \times \frac{99}{365}}$
  - e.  $\$1137 \left( 1 + \frac{0.0975}{12} \right)^2 + \frac{\$2643}{\left( 1 + \frac{0.0975}{12} \right)^3}$
3. **L04** What amount is 62% of \$99?
4. **L04** What amount is 80% of \$156.25?
5. **L04**  $\frac{30}{4}\%$  of \$133. $\overline{33}$  is what amount?
6. **L04** How many minutes is 12.5% of two hours?
7. **L04** The profit forecast for the most recent fiscal quarter is \$23,400. The actual profit is 90% of the forecast profit. What is the actual profit?
8. **L04** Renalda sold Westel stock that she purchased at \$2.20 per share one year ago for a 35% gain. At what price did she sell the stock?
9. **L05** Luther is paid an annual salary of \$56,600 based on a 37.5-hour workweek.
  - a. What is his equivalent hourly wage? (Assume that a year has exactly 52 weeks.)
  - b. What would be his total remuneration for a biweekly pay period of that year if he worked 4.5 hours of overtime at time and a half?
10. **L05** Istvan earns an annual salary of \$61,000 as an executive with a provincial utility. He is paid biweekly. During a strike, he worked 33 hours more than the regular 75 hours for a two-week pay period. What was his gross pay for that period if the company agreed to pay 1.5 times his equivalent hourly rate for overtime? (Assume that a year has exactly 52 weeks.)
11. **L05** Sonja is paid \$42.50 per hour as a veterinarian. She is paid  $1\frac{1}{2}$  times the regular rate for all time exceeding  $7\frac{1}{2}$  hours in a day or  $37\frac{1}{2}$  hours per week. Work on a statutory holiday is paid at double time. What were her gross earnings for a week in which she worked 6, 0, 3,  $7\frac{1}{2}$ , 9,  $7\frac{1}{2}$ , and 8 hours on Saturday to Friday, respectively, and the Monday was a statutory holiday?
12. **L05** Marion receives a monthly base salary of \$1000. On the first \$10,000 of sales above her monthly quota of \$20,000, she is paid a commission of 8%. On any additional sales, the commission rate is 10%. What were her gross earnings for the month of August, in which she had sales amounting to \$38,670?
13. **L05** Lauren's gross pay for July was \$3188.35 on net sales totalling \$88,630. If her base salary is \$1000 per month, what is her rate of commission on sales exceeding her monthly quota of \$40,000?
14. **L05** Havel signed a listing agreement with a real estate agent. The commission rate is 4% on the first \$200,000 of the selling price, and 2.5% on the remainder.
  - a. What commission will Havel pay if he sells his home for \$289,000?
  - b. What is the average commission rate on the selling price?



15. **L06** “Souvenirs and Such” is a gift shop in Niagara Falls. Last year 22% of its revenue came from the sale of clothing, 18% from food items, 32% from novelty items, and the remainder from special services the shop provided for tourists. This past year the shop experienced a 5% increase in the sale of clothing, a 2% increase in the sale of food items, a 9% drop in novelty items, and a 2% drop in special services. What is the shop’s average change in revenue for this year?
16. **L06** Ms. Yong invested a total of \$73,400 in three mutual funds as shown in the following table. The third column shows the change in value of each fund during the subsequent six months.

Mutual fund	Amount Invested (\$)	Change in value (%)
Canadian equity fund	16,800	−4.3
US equity fund	25,600	−1.1
Global equity fund	31,000	8.2

What was the percent change in value of Ms. Yong’s overall mutual fund portfolio during the six-month holding period?

17. **L06** One year ago Helga allocated the funds in her portfolio among five securities in the amounts listed in the following table. The rate of return on each security for the year is given in the third column of the table.

Security	Amount invested (\$)	Rate of return for the year (%)
Company U shares	5000	30
Province V bonds	20,000	−3
Company W shares	8000	−15
Units in Fund X	25,000	13
Company Y shares	4500	45

Calculate the average rate of return for the entire portfolio.

### INTERMEDIATE PROBLEMS

18. **L06** Anthony began the year with \$96,400 already invested in his Snow ’n Ice retail store. He withdrew \$14,200 on March 1 and another \$21,800 on April 1. On August 1, he invested \$23,700, and on November 1, he contributed another \$19,300. What was his average cumulative investment during the year? (Assume that each month has the same length.)
19. **L06** The fiscal year for Pine Valley Skiing Ltd., the owner of a downhill skiing facility, ends on June 30. The company began the recently completed fiscal year with its summer maintenance crew of seven employees. During the fiscal year, employees were hired or laid off on various dates as shown.

Date	Employee changes
September 1	6 hired
November 1	18 hired
December 1	23 hired
March 1	11 laid off
April 1	20 laid off
May 1	16 laid off

What was the average number of employees working for Pine Valley during the fiscal year? (Assume that each month has the same length.)



APPENDIX 1A: THE TEXAS INSTRUMENTS BA II PLUS FORMAT SETTINGS

The calculator allows you to control some aspects of how values are displayed. This section explains how you gain access to and make changes to a list of format settings.

Immediately above the decimal key (in the bottom row of keys) you will find the word **FORMAT**. This position means that the “format” settings are the secondary function of the key. To access the list of settings, press the **2nd** key followed by the decimal key. Hereafter, we will represent this keystroke sequence as **2nd** **Format**.

Press **2nd** **Format**. The calculator’s display now shows:

DEC = n

where  $n$  is one of the digits, 0 to 9. Before we discuss this particular display, you should know that the display is revealing just the first item in the following vertical list:

DEC =	n
DEG	
US	mm-dd-yyyy
US	1,000.
Chn	

You can move down the list using the scroll-down key **↓**. Scroll through the entire list by pressing **↓** four times. Return to the top of the list by pressing **↑** four times or by pressing **↓** one more time.

The first item in the list indicates the number,  $n$ , of decimal places that will be shown in your display. A setting of “9” gives you a floating-decimal display. This means you will see either the *actual* number of decimal places for a terminating decimal, or *nine* decimal places for a non-terminating decimal. (Both alternatives are subject to the limit of 10 digits in the display.) The floating-decimal display is the best general-purpose setting for us. If the current setting is not “9,” change it to “9” by pressing

9 ENTER

Scroll down to the second item in the list (probably appearing as *DEG*). It does not concern us in business math. Scroll down to the third item. The factory setting is

US 12-31-1990

This means that the format for calendar dates is the American format of mm-dd-yyyy (rather than the European format of dd-mm-yyyy). Leave the calculator in the American format and scroll down to the fourth item. The factory setting is

US 1,000.

This means that the calculator is using the American and British convention of employing commas for separators in numbers (rather than the continental European convention of using dots).

Scroll down to the fifth and last item. In the display you will see either “*Chn*” or “*AOS*.” These are alternatives for how the calculator handles the sequence of mathematical operations you enter on the keypad. In the “*Chn*” (chain) calculation method, operations are performed in the sequence you enter them. That is, if you enter

5 + 4 × 2 =

the calculator does the  $5 + 4$  addition before the multiplication, giving a result of 18. (Don’t try it while you are still in the Format list.)



In the “AOS” (algebraic operating system) calculation method, the calculator performs operations according to the standard order of operations presented in [Section 1.1](#). The same calculation then yields the result 13, because the calculator does the multiplication before the addition.

With either “Chn” or “AOS” in the display, you can switch to the other calculation method by pressing **2nd** **SET**. By the **SET** key, we mean the key having SET as its secondary function. The setting you choose is a matter of personal preference. In either case, you can use the calculator’s bracket keys to provide further control over the order of calculations.

Now close and exit from the settings list by pressing **2nd** **QUIT**. By the **QUIT** key, we mean the key having QUIT as its secondary function. The settings you have made will remain (even after the calculator is turned off) until you change them.





## Chapter 2

# Review and Applications of Algebra

### CHAPTER OUTLINE

- 2.1** Operations with Algebraic Expressions
- 2.2** Rules and Properties of Exponents
- 2.3** Solving Linear Equations
- 2.4** Manipulating Equations and Formulas
- 2.5** Solving Word Problems
- 2.6** The Basic Percentage Problem
- 2.7** Percent Change
- \*2.8** Application: Investment Returns from Stocks and Mutual Funds

### LEARNING OBJECTIVES

*After completing this chapter, you will be able to:*

- L01** Simplify algebraic expressions by extracting common factors and applying rules of exponents
- L02** Solve a linear equation in one variable
- L03** Rearrange a formula or equation to isolate a particular variable
- L04** Solve “word problems” that lead to a linear equation in one unknown
- L05** Given any two of the three quantities percent rate, portion, and base, solve for the third
- L06** Solve problems involving percent change
- L07** Calculate the income yield, capital gain yield, and rate of total return on stocks and mutual funds
- L08** Combine rates of total return for successive holding periods



**ALGEBRA IS A BRANCH OF** mathematics in which letters or symbols are used to represent various items (variables). Quantitative relationships (equations) can then be expressed in a concise manner. Algebra has rules and procedures for working with these equations. The rules enable us to calculate values of unknown variables and to derive new relationships among variables.

Algebra is vital for the application of mathematics to business. We (or our computers) make use of equations describing relationships among real-world variables. If your algebraic skills are rusty, you may need to spend more time doing exercises in the first four sections of the chapter than is allotted in your course schedule.

We begin by reviewing basic terminology, techniques for simplifying algebraic expressions, and procedures for solving linear equations. Our refurbished skills will then be employed in word problems and in applications of percent and percent change.



### TIP

#### How to Succeed in Business Mathematics

Since various Business Mathematics and Mathematics of Finance courses start at different points in the book, this Tip appears at the beginning of each of the first four chapters. Connect has a guide entitled “How to Succeed in Business Mathematics.” Read its first two sections (A.1 and A.2) as soon as possible.

## 2.1 Operations with Algebraic Expressions

### Definitions

Business problems that use standard calculations with different numbers for different situations use **algebraic expression** to provide a general solution. The algebraic expression uses letters or symbols as **variables** to represent the quantities that can vary in the solution to the problem.

We will use some simple examples from [Section 1.3](#) to illustrate some basic language of algebra.

Problem	Variables	Algebraic expression
Gross pay is \$11.25 per <i>hour worked</i>	Let $H$ represent the number of <i>hours worked</i>	Gross pay = $\$11.25 \times H$
Gross pay is \$2500 a month plus 4% of <i>sales</i>	Let $S$ represent monthly <i>sales</i>	Gross pay = $\$2500 + 0.04S$
A 5% bonus is awarded on all <i>sales over \$8000</i>	Let $S$ represent total <i>sales</i>	Bonus = $0.05(S - \$8000)$

To obtain a solution to any of the problems above, we simply substitute a specific value for the variable.

The components of an algebraic expression that are separated by addition or subtraction signs are called **terms**.

The expression  $2x$ , containing only one term, is called a **monomial**.

The expression  $2x + 5$  is a **binomial**, containing two terms.

The expression  $3x^2 + 2x + 5$  is a **trinomial** and has three terms.

The name **polynomial** may be used for any expression with more than one term.

Each term in an expression consists of one or more **factors** separated by multiplication or division signs. (Multiplication is implied by writing factors side by side with no multiplication symbol between them.) The numerical factor in a term is called the **numerical coefficient**, and the variable factors together are called the **literal coefficient**.



EXAMPLE 2.1A IDENTIFYING THE TERMS, FACTORS, AND COEFFICIENTS IN A POLYNOMIAL<sup>1</sup>

$3x^2 + xy - 6y^2$  is a trinomial.

Term	Factors	Numerical coefficient	Literal coefficient
$3x^2$	3, $x$ , $x$	3	$x^2$
$xy$	$x$ , $y$	1	$xy$
$-6y^2$	-6, $y$ , $y$	-6	$y^2$

Addition and Subtraction

**L01** Sometimes an algebraic expression may be simplified by adding or subtracting certain terms before any values are substituted for the variables. Terms with the same *literal* coefficients are called **like terms**. Only like terms may be directly added or subtracted. Addition or subtraction of like terms is performed by adding or subtracting their numerical coefficients while keeping their common literal coefficient. For example,  $2xy + 3xy = 5xy$ . Adding or subtracting like terms is often referred to as *collecting* or *combining* like terms.

EXAMPLE 2.1B SIMPLIFYING ALGEBRAIC EXPRESSIONS BY COMBINING LIKE TERMS

- a.  $3a - 4b - 7a + 9b$   
 $= 3a - 7a - 4b + 9b$   
 $= (3 - 7)a + (-4 + 9)b$   
 $= -4a + 5b$

$3a$  and  $-7a$  are like terms;  $-4b$  and  $9b$  are like terms.  
Combine the numerical coefficients of like terms.
- b.  $0.2x + 5x^2 + \frac{x}{4} - x + 3$   
 $= 5x^2 + (0.2 + 0.25 - 1)x + 3$   
 $= 5x^2 - 0.55x + 3$

Convert numerical coefficients to their decimal equivalents.  
Then combine the like terms.
- c.  $\frac{2x}{1.25} - \frac{4}{5} - 1\frac{3}{4}x$   
 $= 1.6x - 0.8 - 1.75x$   
 $= (1.6 - 1.75)x - 0.8$   
 $= -0.15x - 0.8$

Convert numerical coefficients to their decimal equivalents.  
Then combine the like terms.
- d.  $\frac{3x}{1.0164} + 1.049x - x$   
 $= 2.95159x + 1.049x - x$   
 $= (2.95159 + 1.049 - 1)x$   
 $= 3.0006x$

Evaluate the numerical coefficients.  
Combine the like terms.
- e.  $x\left(1 + 0.12 \times \frac{241}{365}\right) + \frac{2x}{1 + 0.12 \times \frac{81}{365}}$   
 $= 1.07923x + \frac{2x}{1.02663}$   
 $= (1.07923 + 1.94812)x$   
 $= 3.02735x$

Evaluate the numerical coefficients.  
Combine the like terms.

<sup>1</sup> Exponents and powers are discussed in Section 2.2. For now, it is sufficient to recall that  $a^2$  means  $a \times a$ ,  $a^3$  means  $a \times a \times a$ , and so on.



## Multiplication and Division

**Multiplication** The product of a monomial and a polynomial is obtained by applying the **distributive property** of multiplication over addition. Every term in the polynomial is multiplied by the monomial. For example:

$$a(b + c) = ab + ac$$

To obtain the product of two polynomials, the distributive property requires the multiplication of *every term* of one polynomial by *every term* of the other polynomial. For example,

$$\begin{aligned}(a + b)(c + d + e) &= a(c + d + e) + b(c + d + e) \\ &= ac + ad + ae + bc + bd + be\end{aligned}$$

After all possible pairs of terms are multiplied, like terms should be combined.

### EXAMPLE 2.1C

### MULTIPLICATION OF ALGEBRAIC EXPRESSIONS

Expand each of the following expressions by carrying out the indicated multiplication.

**a.**  $-x(2x^2 - 3x - 1)$

$$\begin{aligned}&= (-x)(2x^2) + (-x)(-3x) + (-x)(-1) \\ &= -2x^3 + 3x^2 + x\end{aligned}$$

Multiply each term in the trinomial by  $(-x)$ .

The product of two negative quantities is positive.

**b.**  $3m(4m - 6n + 2)$

$$\begin{aligned}&= 3m(4m) + 3m(-6n) + 3m(2) \\ &= 12m^2 - 18mn + 6m\end{aligned}$$

Multiply each term in the trinomial by  $(3m)$ .

In each product, first multiply the numerical coefficients and then multiply the literal coefficients.

**c.**  $2(3a - 2b) - (3b - 2a)$

$$\begin{aligned}&= (2)(3a) + (2)(-2b) + (-1)(3b) + (-1)(-2a) \\ &= 6a - 4b - 3b + 2a \\ &= 8a - 7b\end{aligned}$$

Multiply each term in the first binomial by  $(2)$  and each term in the second binomial by  $(-1)$ .

Combine the like terms.

**d.**  $(7a - 2b)(3b - 2a)$

$$\begin{aligned}&= 7a(3b - 2a) - 2b(3b - 2a) \\ &= 21ab - 14a^2 - 6b^2 + 4ab \\ &= 25ab - 14a^2 - 6b^2\end{aligned}$$

Multiply each term of the first binomial by the second binomial.

Combine the like terms.

**e.**  $(x - 2)(2x^2 - 3x - 4)$

$$\begin{aligned}&= x(2x^2 - 3x - 4) - 2(2x^2 - 3x - 4) \\ &= 2x^3 - 3x^2 - 4x - 4x^2 + 6x + 8 \\ &= 2x^3 - 7x^2 + 2x + 8\end{aligned}$$

Multiply each term of the binomial by the trinomial.

Combine the like terms.

**Division** When dividing a polynomial by a monomial, *each term* of the polynomial must be divided by the monomial.

Although there are a few cases in this book where a monomial or a polynomial is divided by a polynomial, you will not be required to perform the division algebraically.



**TIP****Dividing Out Common Factors**

When dividing out a common factor, remember that *every term* in the numerator and *every term* in the denominator must be divided by that common factor. This is sometimes called “cancelling” common factors. A common error made when dividing a polynomial by a monomial is to cancel one of the factors in the denominator with the same factor in just one of the terms in the numerator. The common factor must be divided out or cancelled in *every term* in the numerator.

Suppose we want to simplify the expression  $\frac{4x + 2}{2}$ .

The denominator 2 must be distributed to every term in the numerator. To illustrate this distribution we could rewrite the expression as:

$$\frac{4x}{2} + \frac{2}{2}$$

Simplifying each term gives us the correct simplified expression  $2x + 1$ .

If you are ever in doubt, substitute a value for  $x$  into both the original and simplified expressions.

If  $x = 5$ , then

$$\frac{4(5) + 2}{2} = \frac{22}{2} = 11 \quad \text{and} \quad 2(5) + 1 = 10 + 1 = 11$$

Since the values are the same, the simplification is correct.

**EXAMPLE 2.1D****DIVISION BY A MONOMIAL**

Simplify each of the following expressions.

a.  $\frac{36x^2y}{60xy^2}$

Identify factors in the numerator and denominator.

$$= \frac{3(12)(x)(x)y}{5(12)(x)(y)y}$$

Divide out factors that appear in both the numerator and the denominator.

$$= \frac{3x}{5y}$$

b.  $\frac{48a^2 - 32ab}{8a}$

Divide each term in the numerator by the denominator.

$$\frac{48a^2}{8a} - \frac{32ab}{8a}$$

Divide out factors that appear in both the numerator and the denominator.

$$= 6a - 4b$$

c.  $\frac{225(1+i)^4}{75(1+i)^2}$

Identify factors that are common to both the numerator and the denominator.

$$= \frac{3(75)(1+i)^2(1+i)^2}{75(1+i)^2}$$

Divide out common factors.

$$= 3(1+i)^2$$

**Substitution**

**Substitution** means assigning a numerical value to each of the algebraic symbols in an expression. Then evaluate the expression by carrying out all the indicated operations. Referring back to our earlier example of the algebraic expression for the gross monthly earnings of the sales staff in the retail store, you can calculate any salesperson's earnings by substituting his or her actual sales figure for  $s$  in the expression

$$\$2500 + 0.04s$$



**EXAMPLE 2.1E****EVALUATING ALGEBRAIC EXPRESSIONS AFTER SUBSTITUTING NUMERICAL VALUES FOR THE VARIABLES**

Evaluate each of the following expressions for the given values of the variables.

- a.  $8p - 9q$  for  $p = 2.5$ ,  $q = -6$
- b.  $3x^2 - 7x - 4$  for  $x = -3$
- c.  $P(1 + rt)$  for  $P = \$100$ ,  $r = 0.09$ ,  $t = \frac{7}{12}$
- d.  $(1 + i)m - 1$  for  $i = 0.05$ ,  $m = 2$
- e.  $\frac{S}{(1 + i)^n}$  for  $S = \$1240$ ,  $i = 0.025$ ,  $n = 4$
- f.  $R\left[\frac{(1 + i)^n - 1}{i}\right]$  for  $R = \$2000$ ,  $i = 0.0225$ ,  $n = 3$

**SOLUTION**

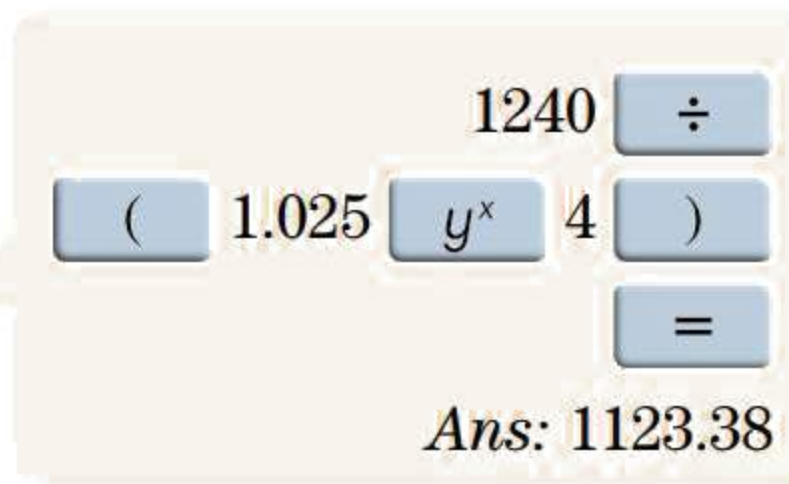
a.  $8p - 9q = 8(2.5) - 9(-6)$  Replace  $p$  by 2.5 and  $q$  by  $-6$ .  
 $= 20 + 54$   
 $= 74$

b.  $3x^2 - 7x - 4 = 3(-3)^2 - 7(-3) - 4$  Replace  $x$  by  $-3$ .  
 $= 3(9) + 21 - 4$   
 $= 27 + 17$   
 $= 44$

c.  $P(1 + rt) = \$100(1 + 0.09 \times \frac{7}{12}) = \$100(1 + 0.0525) = \$105.25$

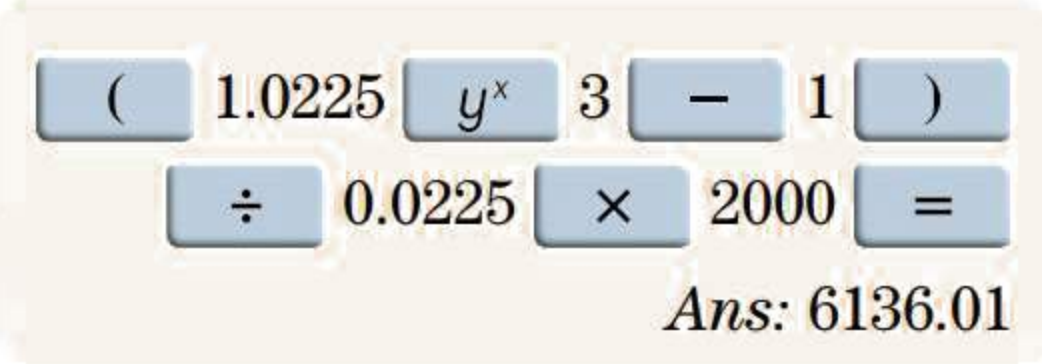
d.  $(1 + i)m - 1 = (1 + 0.05)^2 - 1 = 1.05^2 - 1 = 1.1025 - 1 = 0.1025$

- e. Particularly in later chapters, we will employ callout boxes to present a sequence of keystrokes that may be used on the Texas Instruments BA II PLUS financial calculator for an efficient computation. Here we present this style for the first time.

$$\begin{aligned} \frac{S}{(1 + i)^n} &= \frac{\$1240}{(1 + 0.025)^4} \\ &= \frac{\$1240}{1.025^4} \\ &= \frac{\$1240}{1.103813} \\ &= \$1123.38 \end{aligned}$$


Note that we use a “curly” bracket called a *brace* to indicate the group of mathematical operations performed by the calculator. (You do not need to enter the brackets as indicated in the callout box if your BA II PLUS calculator is set in the AOS mode. See [Appendix 1A](#) regarding this and other FORMAT settings.)

f.  $R\left[\frac{(1 + i)^n - 1}{i}\right] = \$2000\left[\frac{(1 + 0.0225)^3 - 1}{0.0225}\right]$

$$\begin{aligned} &= \$2000\left(\frac{1.0225^3 - 1}{0.0225}\right) \\ &= \$2000\left(\frac{1.0690301 - 1}{0.0225}\right) \\ &= \$2000(3.068006) \\ &= \$6136.01 \end{aligned}$$




**EXERCISE 2.1**

Answers to the odd-numbered problems are at the end of the book.

**CALCULATOR-FREE PROBLEMS**

**Simplify each of the following and collect the like terms.**

- a.  $(-p) + (-3p) + (4p)$
- b.  $5s - 2t - 2s - 4t$
- c.  $4x^2y - 3x^2y + (-5x^2y)$
- d.  $(5s - 2t) - (2s - 4t)$
- e.  $4x^2y + (-3x^2y) - (-5x^2y)$
- f.  $1 - (7e^2 - 5 + 3e - e^3)$
- g.  $(6x^2 - 3xy + 4y^2) - (8y^2 - 10xy - x^2)$
- h.  $(7m^3 - m - 6m^2 + 10) - (5m^3 - 9 + 3m - 2m^2)$
- i.  $2(7x - 3y) - 3(2x - 3y)$
- j.  $4(a^2 - 3a - 4) - 2(5a^2 - a - 6)$
- k.  $15x - [4 - 5x - 6]$
- l.  $6a - [3a - 2b - a]$
- m.  $15x - [4 - 2(5x - 6)]$
- n.  $6a - [3a - 2(2b - a)]$

**Perform the multiplication or division indicated in each of the following expressions and collect the like terms.**

- |   |  |
|---|--|
| o. $4a(3ab - 5a + 6b)$                      | p. $9k(4 - 8k + 7k^2)$                               |
| q. $-5xy(2x^2 - xy - 3y^2)$                 | r. $-(p^2 - 4pq - 5p)\left(\frac{2q}{p}\right)$      |
| s. $(4r - 3t)(2t + 5r)$                     | t. $(3p^2 - 5p)(-4p + 2)$                            |
| u. $(4r - 3t) - (2t + 5r)$                  | v. $(3p^2 - 5p) + (-4p + 2)$                         |
| w. $3(a - 2)(4a + 1) - 5(2a + 3)(a - 7)$    | x. $5(2x - y)(y + 3x) - 6x(x - 5y)$                  |
| y. $\frac{18x^2}{3x}$                       | z. $\frac{6a^2b}{-2ab^2}$                            |
| aa. $\frac{x^2y - xy^2}{xy}$                | bb. $\frac{-4x + 10x^2 - 6x^3}{-0.5x}$               |
| cc. $\frac{12x^3 - 24x^2 + 36x}{48x}$       | dd. $\frac{32a^2b - 8ab + 14ab^2}{2ab}$              |
| ee. $\frac{4a^2b^3 - 6a^3b^2}{2ab^2}$       | ff. $\frac{120(1 + i)^2 + 180(1 + i)^3}{360(1 + i)}$ |
| gg. $\frac{4a^2b^3 - 6ab^2 + 2ab^3}{2ab^3}$ | hh. $\frac{12(1 + i) + 18(1 + i)^2}{36(1 + i)^2}$    |

**BASIC PROBLEMS**

**Evaluate each of the following expressions for the given values of the variables. In Problems 1–11, calculate the result accurate to the nearest cent.**

1.  $3d^2 - 4d + \$15$  for  $d = \$2.50$
2.  $15g - 9h + \$3$  for  $g = \$14$ ,  $h = \$15$
3.  $7x(4y - \$8)$  for  $x = \$3.20$ ,  $y = \$1.50$



4.  $I \div Pr$  for  $P = \$500$ ,  $I = \$13.75$ ,  $r = 0.11$
5.  $\frac{I}{rt}$  for  $r = 0.095$ ,  $I = \$23.21$ ,  $t = \frac{283}{365}$
6.  $\frac{N}{1-d}$  for  $N = \$89.10$ ,  $d = 0.10$
7.  $L(1-d_1)(1-d_2)(1-d_3)$  for  $L = \$490$ ,  $d_1 = 0.125$ ,  $d_2 = 0.15$ ,  $d_3 = 0.05$
8.  $P(1+rt)$  for  $P = \$770$ ,  $r = 0.013$ ,  $t = \frac{223}{365}$
9.  $\frac{S}{1+rt}$  for  $S = \$2500$ ,  $r = 0.085$ ,  $t = \frac{123}{365}$
10.  $\frac{S}{(1+i)^n}$  for  $S = \$850$ ,  $i = 0.0075$ ,  $n = 6$
11.  $P(1+i)^n$  for  $P = \$1280$ ,  $i = 0.025$ ,  $n = 3$

### INTERMEDIATE PROBLEMS

**Simplify each of the following and collect the like terms. Maintain five-figure accuracy in Problems 12–19.**

12.  $\frac{x}{2} - x^2 + \frac{4}{5} - 0.2x^2 - \frac{4}{5}x + \frac{1}{2}$
13.  $\frac{2x+9}{4} - 1.2(x-1)$
14.  $\frac{2x}{1.045} - \frac{2.016x}{3} + \frac{x}{2}$
15.  $\frac{8x}{0.5} + \frac{5.5x}{11} + 0.5(4.6x - 17)$
16.  $y\left(1 - 0.125 \times \frac{213}{365}\right) + \frac{2y}{1 + 0.125 \times \frac{88}{365}}$
17.  $\frac{P}{1 + 0.095 \times \frac{5}{12}} + 2P\left(1 + 0.095 \times \frac{171}{365}\right)$
18.  $\frac{h}{(1 + 0.055)^2} - 3h(1 + 0.055)^3$
19.  $k(1 + 0.04)^2 + \frac{2k}{(1 + 0.04)^2}$

**Evaluate each of the following expressions for the given values of the variables. In Problems 21–24, calculate the result accurate to the nearest cent.**

20.  $(1+i)m - 1$  for  $i = 0.0225$ ,  $m = 4$
21.  $R\left[\frac{(1+i)^n - 1}{i}\right]$  for  $R = \$550$ ,  $i = 0.085$ ,  $n = 3$
22.  $R\left[\frac{(1+i)^n - 1}{i}\right](1+i)$  for  $R = \$910$ ,  $i = 0.1038129$ ,  $n = 4$
23.  $\frac{R}{i}\left[1 - \frac{1}{(1+i)^n}\right]$  for  $R = \$630$ ,  $i = 0.115$ ,  $n = 2$
24.  $P(1+rt_1) + \frac{S}{1+rt_2}$  for  $P = \$470$ ,  $S = \$390$ ,  $r = 0.075$ ,  $t_1 = \frac{104}{365}$ ,  $t_2 = \frac{73}{365}$



## 2.2 Rules and Properties of Exponents

The use of exponents allows us to write algebraic expressions containing repeated factors in a more concise form. If  $n$  is a positive integer, then  $a^n$  is defined by

$$a^n = a \times a \times a \times \dots \times a \text{ (} n \text{ factors)}$$

In this notation,  $a$  is called the **base**,  $n$  is called the **exponent**, and  $a^n$  is read as “ $a$  to the  $n$ ” or “ $a$  to the exponent  $n$ .” The value obtained for  $a^n$  is referred to as “the  $n$ th power of  $a$ ” or sometimes just as “the **power**.” That is,

$$\text{Power} = \text{Base}^{\text{Exponent}}$$

For example,  $4^3 = 4 \times 4 \times 4 = 64$ . In this example, the base is 4, the exponent is 3, and the power is 64. The exponent tells us how many times the base is used as a factor. Note that the power is not the same thing as the exponent—the power is the product resulting from the multiplication. We will use powers extensively in later chapters for compound interest calculations.

### EXAMPLE 2.2A

#### EVALUATING POWERS WITH POSITIVE INTEGRAL EXPONENTS

a.  $3^4 = 3 \times 3 \times 3 \times 3 = 81$

The base is 3, the exponent is 4, and the power is 81. The fourth power of 3 is 81.

b.  $0.1^4 = 0.1 \times 0.1 \times 0.1 \times 0.1 = 0.0001$

c.  $\left(\frac{3}{4}\right)^3 = \left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\left(\frac{3}{4}\right) = \frac{3 \times 3 \times 3}{4 \times 4 \times 4} = \frac{27}{64} = (0.75)(0.75)(0.75) = 0.421875$

d.  $(1.035)^3 = 1.035 \times 1.035 \times 1.035 = 1.108718$

e.  $(-2)^3 = (-2)(-2)(-2) = -8$

An odd power of a negative base is negative.

f.  $(-0.9)^4 = (-0.9)(-0.9)(-0.9)(-0.9) = 0.6561$

An even power of a negative base is positive.



#### TIP

#### The Immediate-Left Trick

Compare  $(-2)^3$  and  $-2^3$ . In which situation is the negative sign part of the base?

We know that  $\text{Power} = \text{Base}^{\text{Exponent}}$ . The base is always to the immediate left of the exponent.

In **Example 2.2A** the powers  $(-2)^3$  and  $(-0.9)^4$  have a bracket to the immediate left of the exponent, meaning that the entire contents of the brackets, including the negative sign, is the base of the power. Without the brackets, the negative sign is NOT part of the base.

$$\begin{aligned} -0.9^4 \text{ also means } -1 \times 0.9^4 &= -1 \times 0.9 \times 0.9 \times 0.9 \times 0.9 = -0.6561, \\ \text{whereas } (-0.9)^4 &= (-0.9) \times (-0.9) \times (-0.9) \times (-0.9) = 0.6561 \end{aligned}$$

The same difference is true for powers such as  $-\left(\frac{2}{3}\right)^3$  compared to  $\left(-\frac{2}{3}\right)^3$  compared to  $-\frac{2^3}{3}$ .

The bases for these powers are  $\frac{2}{3}$ ,  $-\frac{2}{3}$ , and 2, respectively.

A few mathematical operations involving powers occur so frequently that it is convenient to have a set of rules that provide shortcuts. The derivation of the following rules of exponents is straightforward and may be found in any introductory algebra text.



**Rules of Exponents:**

1.  $a^m \times a^n = a^{m+n}$

2.  $\frac{a^m}{a^n} = a^{m-n}$

3.  $(a^m)^n = a^{m \times n}$

4.  $(ab)^n = a^n b^n$

5.  $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

**EXAMPLE 2.2B****USING THE RULES OF EXPONENTS TO SIMPLIFY ALGEBRAIC EXPRESSIONS**

Simplify or evaluate the following expressions.

- a.  $3^2 \times 3^3 = 3^{2+3} = 3^5 = 243$  Rule 1
- b.  $y^5 \times y^4 = y^{5+4} = y^9$  Rule 1
- c.  $(1+i)^6 \times (1+i)^{11} = (1+i)^{6+11} = (1+i)^{17}$  Rule 1
- d.  $\frac{1.01^8}{1.01^5} = 1.01^{8-5} = 1.01^3 = 1.030301$  Rule 2
- e.  $\frac{(1+i)^{20}}{(1+i)^8} = (1+i)^{20-8} = (1+i)^{12}$  Rule 2
- f.  $\frac{x^5 \times x^{14}}{x^9} = x^{5+14-9} = x^{10}$  Rules 1 and 2
- g.  $(k^4)^5 = k^{4 \times 5} = k^{20}$  Rule 3
- h.  $(3^2)^4 = 3^{2 \times 4} = 3^8 = 6561$  Rule 3
- i.  $(5q)^3 = 5^3 q^3 = 125q^3$  Rule 4
- j.  $\left(\frac{0.5}{x}\right)^2 = \frac{0.5^2}{x^2} = \frac{0.25}{x^2}$  Rule 5
- k.  $\left(\frac{3x^6 y^3}{x^2 z^3}\right)^2 = \left(\frac{3x^4 y^3}{z^3}\right)^2$  Rule 2  
 $= \frac{3^2 x^{4 \times 2} y^{3 \times 2}}{z^{3 \times 2}}$  Rules 4 and 5  
 $= \frac{9x^8 y^6}{z^6}$
- l.  $\left(\frac{b^5 - b^3}{b^2}\right)^2 = \left(\frac{b^5}{b^2} - \frac{b^3}{b^2}\right)^2$  Rule 2  
 $= (b^3 - b)^2$

No further simplification is possible using the rules of exponents.

Zero, negative, and fractional exponents have the following rules in order to be consistent with the definition of  $a^n$  and the first three rules of exponents.

**Rules of Zero, Negative, and Fractional Exponents:**

6.  $a^0 = 1$

7.  $a^{-n} = \frac{1}{a^n}$

8.  $a^{1/n} = \sqrt[n]{a}$

9.  $a^{m/n} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$



**TIP****Using the Power Function Key with Negative Exponents**

Press the “sign change” key,  $\boxed{+/-}$ , after you enter the exponent. For example, to evaluate  $1.62^{-5}$ , the sequence of keystrokes is:

1.62  $\boxed{y^x}$  5  $\boxed{+/-}$   $\boxed{=}$

The answer is 0.089624125.

**TIP****Using the Power Function Key with Fractional Exponents**

First calculate the decimal equivalent of the fraction and save it in the calculator’s memory. Later, when you would normally enter the exponent manually, recall its value from the memory. For example, to evaluate  $1.62^{1/6}$ , the sequence of keystrokes is:

1  $\boxed{\div}$  6  $\boxed{=}$   $\boxed{STO}$  1 1.62  $\boxed{y^x}$   $\boxed{RCL}$  1  $\boxed{=}$

Alternatively, you can control the order of operations with brackets. A more efficient keystroke sequence is:

1.62  $\boxed{y^x}$   $\boxed{(}$  1  $\boxed{\div}$  6  $\boxed{)}$   $\boxed{=}$

The answer is 1.0837252.

**EXAMPLE 2.2C****EVALUATING POWERS FOR ZERO, NEGATIVE, AND FRACTIONAL EXPONENTS**

Simplify or evaluate the following. Obtain five-figure accuracy in the answers to parts (f) to (l).

a.  $7.132^0 = 1$  Rule 6

b.  $(0.001)^0 = 1$  Rule 6

c.  $(0.001)^{-1} = \frac{1}{0.001} = 1000$  Rule 7

d.  $(1+i)^{-n} = \frac{1}{(1+i)^n}$  Rule 7

e.  $\left(\frac{x}{y}\right)^{-2} = \frac{1}{\left(\frac{x}{y}\right)^2} = \frac{1}{\frac{x^2}{y^2}} = 1 \times \frac{y^2}{x^2} = \frac{y^2}{x^2} = \left(\frac{y}{x}\right)^2$  Rules 7 and 5

f.  $\left(-\frac{4}{5}\right)^{-2} = \left(-\frac{5}{4}\right)^2 = (-1.25)^2 = 1.5625$  Rule 7

g.  $(1.0125)^{-5} = \left(\frac{1}{1.0125}\right)^5 = (0.987654)^5 = 0.93978$  Rule 7

h.  $(1.0125)^{1/5} = \sqrt[5]{1.0125} = (1.0125)^{0.2} = 1.0025$  Rule 8

i.  $\left(\frac{3}{2}\right)^{3/2} = 1.5^{3/2} = \sqrt{1.5^3} = \sqrt{3.375} = 1.8371 = 1.5^{1.5} = 1.8371$  Rule 9

1.0125  $\boxed{y^x}$   
 $\boxed{(}$  1  $\boxed{\div}$  5  $\boxed{)}$   $\boxed{=}$   
 Ans: 1.0025



j.  $\$321(1 + 0.025)^{-8} = \$321(0.820747) = \$263.46$

Rule 7

1.025  $y^x$  8  $+/-$   
 $\times$  321  $=$   
 Ans: 263.46

k.  $(2.025)^{-(1/4)} = \left(\frac{1}{2.025}\right)^{(1/4)} = (0.493827)^{0.25} = 0.83829$

Rules 7 and 8

l.  $\frac{1 - (1.025)^{-30}}{0.025} = \frac{1 - 0.476743}{0.025} = 20.930$

Rule 7

Assuming the "AOS" calculation setting (see Appendix 1A)

( 1 - 1.025  $y^x$  30  
 $+/-$  )  $\div$  0.025  $=$   
 Ans: 20.930

## EXERCISE 2.2

Answers to the odd-numbered problems are at the end of the book.

### CALCULATOR-FREE PROBLEMS

Simplify each of the following.

a.  $a^2 \times a^3$

b.  $(x^6)(x^{-4})$

c.  $b^{10} \div b^6$

d.  $h^7 \div h^{-4}$

e.  $(1 + i)^4 \times (1 + i)^9$

f.  $(1 + i) \times (1 + i)^n$

g.  $(x^4)^7$

h.  $(y^3)^3$

i.  $(t^6)^{1/3}$

j.  $(n^{0.5})^8$

k.  $\frac{(x^5)(x^6)}{x^9}$

l.  $\frac{(x^5)^6}{x^9}$

m.  $[2(1 + i)]^2$

n.  $\left(\frac{1 + i}{3i}\right)^3$

o.  $\frac{4r^5t^6}{(2r^2t)^3}$

p.  $\frac{(-r^3)(2r)^4}{(2r^{-2})^2}$

### BASIC PROBLEMS

Evaluate each of the following expressions to six-figure accuracy.

1.  $8^{4/3}$

2.  $-27^{2/3}$

3.  $7^{3/2}$

4.  $5^{-3/4}$

5.  $(0.001)^{-2}$

6.  $0.893^{-1/2}$

7.  $(1.0085)^5 (1.0085)^3$

8.  $(1.005)^3 (1.005)^{-6}$

9.  $\sqrt[3]{1.03}$

10.  $\sqrt[6]{1.05}$

### INTERMEDIATE PROBLEMS

Evaluate each of the following expressions to six-figure accuracy.

11.  $(4^4)(3^{-3})\left(-\frac{3}{4}\right)^3$

12.  $\left[\left(-\frac{3}{4}\right)^2\right]^{-2}$

13.  $\left(\frac{2}{3}\right)^3 \left(-\frac{3}{2}\right)^2 \left(-\frac{3}{2}\right)^{-3}$

14.  $\left(-\frac{2}{3}\right)^3 \div \left(\frac{3}{2}\right)^{-2}$

15.  $\frac{1.03^{16} - 1}{0.03}$

16.  $\frac{(1.0083)^{30} - 1}{0.0083}$

17.  $\frac{1 - 1.0225^{-20}}{0.0225}$

18.  $\frac{1 - (1.006)^{-32}}{0.006}$

19.  $(1 + 0.0275)^{1/3}$

20.  $(1 + 0.055)^{1/6} - 1$



## 2.3 Solving Linear Equations

Most of the applications and problems in this book will result in an equation containing a single variable or unknown. If this variable's exponent is 1, the equation is a **linear equation**. If the variable appears with an exponent other than 1, the equation is a **nonlinear equation**.

### EXAMPLE 2.3A

#### EXAMPLES OF LINEAR AND NONLINEAR EQUATIONS

- $3x - 7 = 5 - 9x$  is a linear equation.
- $x^2 - x = 12$  is a nonlinear equation because  $x$  has an exponent different from 1 in one of the terms.
- $\$150(1 + i)^4 = \$219.62$  is a nonlinear equation because of the presence of terms in  $i^4$ ,  $i^3$ , and  $i^2$  when  $(1 + i)^4$  is expanded.
- $2x = 32$  is a nonlinear equation because  $x$  is in the exponent.

### Solving a Linear Equation in One Unknown

**L02** A **root** or solution of an equation is any numerical value of the variable that makes the two sides of the equation equal; it makes the equation true. For example,  $x = 1$  is a root of  $3x - 7 = 5 - 9x$  because both sides have the same value ( $-4$ ) when you substitute  $x = 1$ .

A linear equation in one variable has only one root; a nonlinear equation may have more than one root. The process of determining the root or roots of the equation is called *solving the equation*. The procedure for solving a linear equation involves three steps:

- Separate like terms, leaving terms containing the variable on one side of the equation and the remaining terms on the other side of the equation.
- Combine the like terms on each side of the equation.
- Obtain the root or solution by dividing both sides of the equation by the numerical coefficient of the variable.

After you calculate the root of an equation, you can verify that it is a root by substituting it in the original equation. The root is correct (and is said to *satisfy* the equation) if the resulting values of both sides of the equation are equal.

### EXAMPLE 2.3B

#### SOLVING LINEAR EQUATIONS IN ONE UNKNOWN

Solve the following equations and verify the solutions.

- $8x - 11 = 5x + 4$
- $0.5x - 0.75 + 7x = 3x + 1.5$
- $5(x - 4) + 2 = 2(x + 2) - 1$

#### SOLUTION

a.  $8x - 11 = 5x + 4$

$$\begin{aligned} 8x - 11 - 5x &= 5x + 4 - 5x \\ 3x - 11 &= 4 \end{aligned}$$

$$\begin{aligned} 3x - 11 + 11 &= 4 + 11 \\ 3x &= 15 \end{aligned}$$

$$\begin{aligned} \frac{3x}{3} &= \frac{15}{3} \\ x &= 5 \end{aligned}$$

Subtract  $5x$  from both sides so that terms in  $x$  will be on the left side only.

Note that moving  $5x$  from the right side to the left side with a change of sign also produces this result. It is more efficient to use this shortcut (called *transposition*).

Add 11 to both sides so that numerical terms will be on the right side only.

Transposing the term “ $-11$ ” with a change of sign from the left side to the right side produces this same result.

Divide both sides by the numerical coefficient of  $x$ .



*Verification:*

$$\begin{aligned}\text{Left-hand side (LHS)} &= 8x - 11 \\ &= 8(5) - 11 = 29\end{aligned}$$

$$\begin{aligned}\text{Right-hand side (RHS)} &= 5x + 4 \\ &= 5(5) + 4 = 29\end{aligned}$$

Since LHS = RHS,  $x = 5$  is the root, or solution.

**b.**  $0.5x - 0.75 + 7x = 3x + 1.5$       Transpose  $3x$  to the LHS and  $-0.75$  to the RHS, and change their signs.  
 $0.5x + 7x - 3x = 1.5 + 0.75$   
 $4.5x = 2.25$       Divide both sides by the numerical coefficient of  $x$ .  
 $x = \frac{2.25}{4.5}$   
 $x = 0.5$

*Verification:*

$$\text{LHS} = 0.5(0.5) - 0.75 + 7(0.5) = 0.25 - 0.75 + 3.5 = 3.0$$

$$\text{RHS} = 3(0.5) + 1.5 = 1.5 + 1.5 = 3.0$$

Since LHS = RHS,  $x = 0.5$  is the solution.

**c.**  $5(x - 4) + 2 = 2(x + 2) - 1$   
 $5x - 20 + 2 = 2x + 4 - 1$       Expand each side using the distributive property of multiplication over addition.  
 $5x - 18 - 2x = 3$       Transpose  $2x$  to the LHS and add the numerical terms to simplify.  
 $3x = 3 + 18$       Collect like terms on the LHS and transpose  $-18$  to the RHS.  
 $\frac{3x}{3} = \frac{21}{3}$       Divide both sides by 3.  
 $x = 7$

*Verification:*

$$\text{LHS} = 5(7 - 4) + 2 = 5(3) + 2 = 15 + 2 = 17$$

$$\text{RHS} = 2(7 + 2) - 1 = 2(9) - 1 = 18 - 1 = 17$$

Since LHS = RHS,  $x = 7$  is the solution.

**EXAMPLE 2.3C****SOLVING LINEAR EQUATIONS HAVING FRACTION COEFFICIENTS**

Solve each of the following equations for  $x$  (accurate to two decimal places). Verify the solution.

**a.**  $\frac{1}{4}x - \frac{2}{5} = \frac{1}{2}x + \frac{3}{10}$

**b.**  $\frac{x}{1 + 0.11 \times \frac{75}{365}} + 2x \left( 1 + 0.11 \times \frac{92}{365} \right) = \$1150.96$

**SOLUTION**

- a.** In this equation, the numerical coefficients are fractions; these coefficients should be changed to decimal form. Then carry on with the usual procedure for solving a linear equation.

$$\begin{aligned}\frac{1}{4}x - \frac{2}{5} &= \frac{1}{2}x + \frac{3}{10} \\ 0.25x - 0.4 &= 0.5x + 0.3 \\ 0.25x - 0.5x &= 0.3 + 0.4 \\ -0.25x &= 0.7 \\ x &= -\frac{0.7}{0.25} \\ x &= -2.8\end{aligned}$$

*Verification:*

$$\text{LHS} = \frac{1}{4}(-2.8) - \frac{2}{5} = 0.25(-2.8) - 0.4 = -1.1$$

$$\text{RHS} = \frac{1}{2}(-2.8) + \frac{3}{10} = 0.5(-2.8) + 0.3 = -1.1$$

Since LHS = RHS,  $x = -1.1$  is the solution.



- b. In this equation, the numerical coefficients are not simple numbers as in the previous examples. These coefficients should be reduced to a single number (which may not be an integer) by performing the indicated operations. Then carry on with the usual three-step procedure for solving a linear equation in one unknown.

$$\begin{aligned}\frac{x}{1.0226027} + 2x(1.0277260) &= \$1150.96 \\ 0.9778969x + 2.055452x &= \$1150.96 \\ 3.033349x &= \$1150.96 \\ x &= \frac{\$1150.96}{3.033349} \\ x &= \$379.44\end{aligned}$$

Verification:

$$\begin{aligned}\text{LHS} &= \frac{\$379.44}{1 + 0.11 \times \frac{75}{365}} + 2(\$379.44) \left(1 + 0.11 \times \frac{92}{365}\right) \\ &= \frac{\$379.44}{1.0226027} + 2(\$379.44)(1.0277260) \\ &= \$371.05 + \$779.92 \\ &= \$1150.97 \\ &= \text{RHS}\end{aligned}$$

#### BA II PLUS Keystrokes

(when set for “AOS” calculations)

**Step 1:** Obtain the coefficient of  $x$  in the first term and save in memory.

1  $\div$  0.11  $\times$  75  $\div$  365  
=  $\frac{1}{1.0226027}$  STO 1

**Step 2:** Obtain the coefficient of  $x$  in the second term.

2  $\times$  ( 1  $\div$  0.11  $\times$  92  
 $\div$  365 ) =

**Step 3:** Recall and add the first term's coefficient. Save this sum.

+ RCL 1 = STO 2

**Step 4:** Divide the right side by this saved combined coefficient of  $x$ .

1150.96  $\div$  RCL 2 =

Ans: 379.44

The \$0.01 difference between the LHS and the RHS arises from rounding the solution  $x = \$379.4354$  to the nearest cent and then using the rounded value for the verification.

## APP 4 THAT

To find a quick way either to solve an equation or to gain further understanding on how to solve an equation in your homework problems, search the App Store on your tablet, smartphone, or smart watch using the key words **EQUATION SOLVER**.

You will find many free and paid apps that generate step-by-step solutions to guide you through the solving process either by typing in the equation or by taking a photo of the equation to be solved.

### EXERCISE 2.3

Answers to the odd-numbered problems are at the end of the book.

#### CALCULATOR-FREE PROBLEMS

Solve the following equations.

a.  $2a - 9 = a + 1$

c.  $6(y - 4) = 0$

e.  $-7x - 10 = 11$

g.  $12 - 3x - 8 = -10x - 10$

i.  $\frac{x+2}{x-1} = 4$

b.  $5 + 3x = 20$

d.  $\frac{b}{3} - 1 = 5$

f.  $4 = \frac{10-m}{3}$

h.  $2(4x - 5) = x - 3$

j.  $\frac{15p}{2} = 7p + 4$



**BASIC PROBLEMS****Solve the following equations.**

1.  $10a + 10 = 12 + 9a$
2.  $29 - 4y = 2y - 7$
3.  $0.5(x - 3) = 20$
4.  $\frac{1}{3}(x - 2) = 4$
5.  $y = 192 + 0.04y$
6.  $x - 0.025x = 341.25$
7.  $12x - 4(2x - 1) = 6(x + 1) - 3$
8.  $3y - 4 = 3(y + 6) - 2(y + 3)$
9.  $8 - 0.5(x + 3) = 0.25(x - 1)$
10.  $5(2 - c) = 10(2c - 4) - 6(3c + 1)$
11.  $3.1t + 145 = 10 + 7.6t$
12.  $1.25y - 20.5 = 0.5y - 11.5$
13.  $\frac{x+2}{5} = x + 0.8$
14.  $\frac{3.5}{x-1} = 2.5$

**INTERMEDIATE PROBLEMS****Solve the following equations. The solutions to Problems 17–24 should be accurate to the cent.**

15.  $\frac{10a}{2.2} + (2.2)^2 = 6 + a(2.2)^3$
16.  $21 - \frac{b}{1.45} = 5.5b - 9$
17.  $(1.065)^2x - \frac{x}{1.065} = \$35$
18.  $12x - 4x(1.06)^4 = \$1800$
19.  $\frac{x}{1.1^2} + 2x(1.1)^3 = \$1000$
20.  $\frac{3x}{1.025^6} + x(1.025)^8 = \$2641.35$
21.  $\frac{2x}{1.03^7} + x + x(1.03)^{10} = \$1000 + \frac{\$2000}{1.03^4}$
22.  $x(1.05)^3 + \$1000 + \frac{x}{1.05^7} = \frac{\$5000}{1.05^2}$
23.  $x\left(1 + 0.095 \times \frac{84}{365}\right) + \frac{2x}{1 + 0.095 \times \frac{108}{365}} = \$1160.20$
24.  $\frac{x}{1 + 0.115 \times \frac{78}{365}} + 3x\left(1 + 0.115 \times \frac{121}{365}\right) = \$1000\left(1 + 0.115 \times \frac{43}{365}\right)$



## 2.4 Manipulating Equations and Formulas

An **equation** is a statement that one algebraic expression equals another algebraic expression. It represents a precise relationship among the variables and numbers. When the relationship described by an equation is a general one with broad application, we usually refer to the equation as a *formula*.

Most of the important formulas we will develop and use in this book are listed in the Summary of Key Formulas section on Connect. Some of them are pre-programmed into your financial calculator.

Keep in mind that every one of these formulas is a shorthand description of a real-world relationship in business or finance. Some of the formulas are amazingly powerful.

**L03** In a typical application, you will decide that a particular formula applies to the problem or situation at hand. From the given information you will be able to determine numerical values for all but one of the variables in the formula. If the remaining unknown variable is embedded in the right-hand side, you will have to manipulate (rearrange) the formula by a series of steps designed to isolate the unknown variable.

Let us use Formula (6-2) to describe two general approaches to manipulating formulas. At this point, it does not matter if you have absolutely no idea what the symbols  $S$ ,  $P$ ,  $r$ , and  $t$  represent in the formula

$$S = P(1 + rt) \quad (6-2)$$

This formula has four variables:  $S$ ,  $P$ ,  $r$ , and  $t$ . To be able to solve for any one of these, you need to know values for the other three variables.

Suppose you are given values for  $S$ ,  $P$ , and  $t$ , and you must solve for  $r$ . You can choose to do either of the following:

- *First substitute* the numerical values for  $S$ ,  $P$ , and  $t$ , and *then manipulate (or rearrange)* the equation to solve for  $r$ ; **or**
- *First rearrange* the formula to isolate  $r$ , and *then substitute* the numerical values for  $S$ ,  $P$ , and  $t$ .

Normally the first approach is easier because, after substitution, you can usually simplify the right-hand side to some degree. The second approach is more useful in situations when the rearrangement is going to be used over and over again.

Regardless of which approach you take for manipulating a formula or equation,<sup>2</sup> the same fundamental principle applies at every step:

*Both sides of an equation must be treated in exactly the same way.*

For example,

- You can add the same number or variable to *both* sides;
- You can subtract the same number or variable from *both* sides;
- You can multiply *both* sides by the same number or variable;
- You can divide *both* sides by the same number or variable;
- You can raise *both* sides to the same exponent. This means, for example, that you can cube both sides (that is, raise both sides to the exponent “3”), or take the square root of both sides (that is, raise both sides to the exponent “ $\frac{1}{2}$ ”).

<sup>2</sup> When we substitute numerical values for some of the variables in a formula, we are applying a general formula to a particular situation. At this point, we usually start referring to the mathematical relationship simply as an equation because of its more limited applicability.



**EXAMPLE 2.4A****DEMONSTRATING THE TWO APPROACHES FOR FORMULA MANIPULATION**

Suppose the formula  $S = P(1 + rt)$  applies to a situation for which  $S = \$1040$ ,  $P = \$1000$ , and  $t = 0.5$ .

- Calculate  $r$  by first substituting the given values into the formula and then rearranging the equation.
- Calculate  $r$  by first rearranging the general formula to isolate  $r$ , and then substituting numerical values for  $S$ ,  $P$ , and  $t$ .

**SOLUTION**

- Substitute the values  $S = \$1040$ ,  $P = \$1000$ , and  $t = 0.5$  into  $S = P(1 + rt)$ , giving

$$\$1040 = \$1000[1 + r(0.5)]$$

$$\$1040 = \$1000(1 + 0.5r)$$

$$\$1040 = \$1000 + \$500r \quad \text{after multiplying both "1" and "0.5r" by "\$1000"}$$

To leave only the term containing  $r$  on the right side, subtract  $\$1000$  from *both* sides.

$$\$1040 - \$1000 = \$1000 + \$500r - \$1000$$

$$\$40 = \$500r$$

At the end, we want  $r$  by itself on the left-hand side. You can simply interchange the left and right sides of an equation—it does not change the equality.

$$\$500r = \$40$$

Finally, divide *both* sides by “ $\$500$ ” to make  $r$ ’s coefficient “1” instead of “ $\$500$ .”

$$\frac{\$500r}{\$500} = \frac{\$40}{\$500}$$

$$r = 0.08$$

- We want an equivalent formula with  $r$  rather than  $S$  isolated on the left side. Interchange the two sides of  $S = P(1 + rt)$ , giving

$$P(1 + rt) = S$$

Next multiply both “1” and “ $rt$ ” by “ $P$ .” These steps yield

$$P + Prt = S$$

To have only the term containing  $r$  remaining on the left side, subtract  $P$  from both sides.

$$P + Prt - P = S - P$$

$$Prt = S - P$$

Divide both sides by  $Pt$  to eliminate these variables from the left-hand side.

$$\frac{Prt}{Pt} = \frac{S - P}{Pt}$$

(You may draw a slash through the factors  $P$  and  $t$  in the numerator and denominator on the left-hand side to signify that these variables “cancel out.” But note that you cannot “cancel” the  $P$ s on the right-hand side because  $P$  does not appear in *both* terms of the numerator.)

$$r = \frac{S - P}{Pt}$$

Now substitute the values for  $S$ ,  $P$ , and  $t$ , giving

$$r = \frac{\$1040 - \$1000}{\$1000 \times 0.5} = \frac{\$40}{\$500} = 0.08$$



**TIP****A Shortcut**

We obtained the intermediate outcome

$$Prt = S - P$$

in part (b) of **Example 2.4A** by subtracting  $P$  from *both* sides of  $P + Prt = S$ . The same outcome may be achieved by moving the term  $P$  with a *change of sign* to the other side of the equation.

Most people prefer the second approach, particularly in cases where they want to shift two or three terms at the same time. Hereafter, we will employ this shortcut (sometimes called *transposing a term*). Note that for a term like  $Prt$ , you cannot transpose just the factor  $P$ —all numerical and algebraic factors in a term may be transposed only as a complete “package.”

**EXAMPLE 2.4B****DEMONSTRATING THE TWO APPROACHES FOR FORMULA MANIPULATION**

In a particular application of the formula  $c = \frac{V_f - V_i}{V_i}$ , the values for  $c$  and  $V_f$  are 0.15 and \$230, respectively.

- Obtain  $V_i$  by first substituting the given values into the formula and then rearranging the equation.
- Obtain  $V_i$  by first rearranging the general formula to isolate  $V_i$ , and then substituting numerical values for  $c$  and  $V_f$ .

**SOLUTION**

- Substitute  $c = 0.15$  and  $V_f = \$230$  into

$$c = \frac{V_f - V_i}{V_i}$$

giving

$$0.15 = \frac{\$230 - V_i}{V_i}$$

Multiply both sides by  $V_i$  to get rid of the fraction on the right side.

$$\begin{aligned} 0.15V_i &= \frac{\$230 - V_i}{V_i} \times V_i \\ 0.15V_i &= \$230 - V_i \end{aligned}$$

We want all terms containing  $V_i$  to be on the left side. Move the term  $V_i$  (with a change of sign) to the left side.

$$V_i + 0.15V_i = \$230$$

Remember that  $V_i$  really means  $1V_i$ . Therefore, we can add the two terms on the left side, giving

$$1.15V_i = \$230$$

Divide both sides by 1.15 to make  $V_i$ 's coefficient “1.”

$$\begin{aligned} \frac{1.15V_i}{1.15} &= \frac{\$230}{1.15} \\ V_i &= \$200.00 \end{aligned}$$

- We want an equivalent formula with  $V_i$  isolated on the left side. To get rid of the denominator on the right side, multiply both sides by  $V_i$ .

$$c(V_i) = \frac{V_f - V_i}{V_i} \times V_i$$

Now the right-hand side simplifies to

$$cV_i = V_f - V_i$$

Transpose  $V_i$  to the left side in order to collect all terms containing  $V_i$  on the left side.

$$V_i + cV_i = V_f$$



Now extract the common factor  $V_i$  on the left side.

$$V_i(1 + c) = V_f$$

The final step to isolate  $V_i$  is to divide both sides by  $(1 + c)$ . This will cancel the  $(1 + c)$  factor on the left side, leaving

$$V_i = \frac{V_f}{1 + c}$$

If we now substitute  $V_f = \$230$  and  $c = 0.15$ , we get

$$V_i = \frac{\$230}{1 + 0.15} = \frac{\$230}{1.15} = \$200.00$$



### TIP

#### Another Shortcut

We obtained the intermediate outcome

$$0.15V_i = \$230 - V_i$$

in part (a) of **Example 2.4B** by multiplying both sides of the equation

$$0.15 = \frac{\$230 - V_i}{V_i}$$

by  $V_i$ . The same outcome may be achieved by removing  $V_i$  from the denominator on the right side and multiplying it against the numerator on the left side of the equation.

Most people prefer this second approach (commonly called *cross-multiplication*), particularly when both sides of an equation are fractions. For example, to isolate  $x$  in the equation

$$\frac{a}{b} = \frac{c}{x}$$

you can first cross-multiply to obtain

$$ax = cb$$

Then divide both sides by  $a$ . This gives

$$x = \frac{cb}{a}$$

## EXERCISE 2.4

Answers to the odd-numbered problems are at the end of the book.

### BASIC PROBLEMS

1. Use Formula (6-1),  $I = Prt$ , to calculate  $P$  if  $r = 0.05$ ,  $I = \$6.25$ , and  $t = 0.25$ .
2. Use Formula (13-1),  $PV = \frac{PMT}{i}$ , to calculate  $i$  if  $PMT = \$900$  and  $PV = \$150,000$ . (There are several instances in our formulas where a two- or three-letter symbol is used for a variable. This is usually done to make the symbol more suggestive of the quantity it represents. For example, we use  $PMT$  to represent the amount of each payment in a series of regular payments. The symbol  $P$  has already been taken to represent another quantity that begins with “p.”)
3. Use Formula (6-2),  $S = P(1 + rt)$ , to calculate  $P$  if  $r = 0.004$ ,  $S = \$3626$ , and  $t = 9$ .
4. Use Formula (4-1),  $N = L(1 - d)$ , to calculate  $L$  if  $N = \$891$  and  $d = 0.10$ .
5. Use Formula (4-1),  $N = L(1 - d)$ , to calculate  $d$  if  $N = \$410.85$  and  $L = \$498.00$ .
6. Use Formula (6-2),  $S = P(1 + rt)$ , to calculate  $t$  if  $r = 0.0025$ ,  $S = \$5100$ , and  $P = \$5000$ .
7. Formula (5-5),  $NI = (CM)X - FC$ , contains three two-letter symbols. Use it to calculate  $CM$  if  $NI = \$15,000$ ,  $X = 5000$ , and  $FC = \$60,000$ .



8. Use Formula (5-5),  $NI = (CM)X - FC$ , to obtain  $X$  if  $NI = -\$542.50$ ,  $CM = \$13.50$ , and  $FC = \$18,970$ .
9. Use Formula (4-2),  $N = L(1 - d_1)(1 - d_2)(1 - d_3)$ , to calculate  $L$  if  $N = \$1468.80$ ,  $d_1 = 0.20$ ,  $d_2 = 0.15$ , and  $d_3 = 0.10$ .
10. Use Formula (4-2),  $N = L(1 - d_1)(1 - d_2)(1 - d_3)$ , to calculate  $d_2$  if  $N = \$70.29$ ,  $L = \$99.99$ ,  $d_1 = 0.20$ , and  $d_3 = 0.05$ .
11. Use the formula  $FV = PV(1 + i_1)(1 + i_2)(1 + i_3)$  to determine  $i_1$  if  $PV = \$1000$ ,  $FV = \$1094.83$ ,  $i_2 = 0.03$ , and  $i_3 = 0.035$ .
12. Use Formula (10-1),  $FV = PMT \left[ \frac{(1 + i)^n - 1}{i} \right]$ , to obtain  $PMT$  if  $FV = \$1508.54$ ,  $n = 4$ , and  $i = 0.05$ .
13. Use Formula (10-2),  $PV = PMT \left[ \frac{1 - (1 + i)^{-n}}{i} \right]$ , to obtain  $PMT$  if  $PV = \$6595.20$ ,  $n = 20$ , and  $i = 0.06$ .
14. Rearrange Formula (6-1),  $I = Prt$ , to isolate  $t$  on the left side.
15. Rearrange Formula (13-1),  $PV = \frac{PMT}{i}$ , to isolate  $i$  on the left side.
16. Rearrange Formula (4-1),  $N = L(1 - d)$ , to isolate  $d$  on the left side.
17. Rearrange Formula (5-5),  $NI = (CM)X - FC$ , to isolate  $CM$  on the left side.
18. Rearrange Formula (5-5),  $NI = (CM)X - FC$ , to isolate  $X$  on the left side.
19. Rearrange Formula (6-2),  $S = P(1 + rt)$ , to isolate  $r$  on the left side.
20. Rearrange Formula (6-2),  $S = P(1 + rt)$ , to isolate  $t$  on the left side.
21. Rearrange Formula (4-2),  $N = L(1 - d_1)(1 - d_2)(1 - d_3)$ , to isolate  $d_1$  on the left side.
22. Rearrange Formula (4-2),  $N = L(1 - d_1)(1 - d_2)(1 - d_3)$ , to isolate  $d_3$  on the left side.
23. Rearrange the formula  $FV = PV(1 + i)^n$  to isolate  $PV$  on the left side.

### INTERMEDIATE PROBLEMS

24. Use the formula  $FV = PV(1 + i)^n$  to calculate  $i$  if  $PV = \$2000$ ,  $FV = \$9321.91$ , and  $n = 20$ .
25. Use the formula  $PV = FV(1 + i)^{-n}$  to calculate  $i$  if  $PV = \$5167.20$ ,  $FV = \$10,000$ , and  $n = 15$ .
26. Rearrange the formula  $FV = PV(1 + i)^n$  to isolate  $i$  on the left side.

## 2.5 Solving Word Problems

**L04** In the preceding sections, we reviewed procedures for solving linear equations and working with formulas. With practice, solving linear equations becomes a mechanical procedure that follows a fairly routine series of steps. However, *practical* applications of mathematics rarely come as given equations that need only to be solved. Instead, a problem is presented to us in a more informal descriptive manner. We must deduce mathematical relationships from the given information and from our broader knowledge of general concepts and principles.

Constructing algebraic equations from given information cannot be reduced to mechanical steps (such as those previously listed for solving linear equations). We can, however, outline a *general approach* for solving word problems.

### A General Approach for Solving Word Problems

Particularly if you are having difficulty with a word problem, use the following procedure to reduce the solution to small steps. Let us assume for now that there is only one unknown in the problem.



**Step 1:** *Read the entire problem* to gain a sense of the topic involved and what is being asked. For example, you might find that the problem involves a loan repayment and you are asked to determine the size of the monthly payment.

**Step 2:** Now take pencil and paper in hand. On a second reading, *extract and label the given data*. *Identify the unknown quantity and choose a symbol for it. Draw and label a diagram if appropriate.* There are standard symbols for many quantities. These should be used to label numerical values as they are extracted. Otherwise, use one or two words to identify each value. Choose a symbol to represent the unknown quantity. Diagrams are particularly useful in problems involving multiple payments over a period of time. Incorporate as much information as possible in the diagram.

**Step 3:** *Create a word equation* that relates what you know (the data) to what you want to know (the unknown quantity). The word equation may be based on a fundamental principle of broad application, or it may be a unique relationship stated or implied in the problem itself. An example of a word equation is

$$\text{Profit} = \text{Revenues} - \text{Expenses}$$

**Step 4:** *Convert the word equation to an algebraic equation.* Express the words and phrases in the word equation in terms of the unknown quantity's symbol and the numerical values extracted in Step 2.

**Step 5:** *Solve the equation.* Check that the answer is reasonable. Write a concluding statement that directly responds to the question asked.

### Summary of the Steps for Solving Word Problems

1. Read the entire problem.
2. Extract and label the data. Identify the unknown quantity and specify its symbol. Draw and label a diagram if appropriate.
3. Create a word equation that relates the given data to the unknown quantity.
4. Convert the word equation to an algebraic equation.
5. Solve the equation.

To remember the steps, think of “REDWAS” representing **R**ead, **E**xtract and **D**raw, **W**ord equation, **A**lgebraic equation, and **S**olve.



### TIP

### Overcoming That Old “I-Just-Can’t-Seem-to-Get-Started” Feeling

Have you ever stared and stared at a word problem without getting anywhere? (If you have, you belong to a rather large club.) Think about how you have approached word problems in the past. Did you begin by trying to find a formula in which to substitute the given numbers? If so, you were really trying to do Steps 1, 2, 3, and 4 all at once! It's not surprising that you became stumped on problems of even moderate difficulty. Even if you happen to hit on the right formula and calculate the right answer, the “formula-browsing” approach is not an effective way to “do” mathematics. It omits Step 3, where you bring mathematical ideas and concepts to bear on a particular situation. For problems of any complexity, you should use our five-step ladder instead of trying to leap tall problems in a single bound.

The following examples illustrate the five-step approach for solving word problems.



**EXAMPLE 2.5A THE STEP-BY-STEP APPROACH TO SOLVING A WORD PROBLEM**

- a. Elsie wants to make \$645 selling her pottery mugs at the local craft fair. The fee to rent a table is \$75 and she plans on charging \$12 for her mugs. How many mugs does she have to sell to reach her goal?
- b. Elsie makes two sizes of mugs to sell. The number of large mugs she has is twice as many as the number of smaller mugs. If she charges \$14 for the large mugs and \$12 for the small mugs, what is the minimum number of each size she needs to sell to reach her goal?

**SOLUTION**

- a. **Step 1:** Read the problem. (This step is assumed hereafter.)

**Step 2:** Extract and label the data. Identify the unknown and define its symbol.

$$\text{Goal (Profit)} = \$645 \quad \text{Price} = \$12 \text{ per mug} \quad \text{Rental fee (Expenses)} = \$75$$

Since the problem is asking for the number of mugs, we will let  $x$  represent the number of mugs.

Don't worry if you are not sure what to call values. A short description works just as well and can be refined in Step 3.

**Step 3:** Create a word equation that relates the given data to the unknown. This relation will often be described in the problem or you may have to use information you have learned in your course work. The given problem does not spell out the relationship between Elsie's goal and her price and costs, but we can generally state that:

$$\text{Elsie's goal} = \text{Money in} - \text{Money out}$$

In terms of this problem, we can refine this to:

$$\text{Profit} = \text{Revenue} - \text{Expenses}$$

$$\text{Profit} = (\text{Price} \times \text{Number of mugs sold}) - \text{Expenses}$$

**Step 4:** Convert the word equation to an algebraic equation, substituting in the known data.

$$\$645 = \$12x - \$75$$

**Step 5:** Solve the equation.

$$\$645 = \$12x - \$75$$

$$x = 60$$

Elsie needs to sell 60 mugs to reach her goal of making \$645.

Check:  $(\$12 \times 60 - \$75 = \$645)$

- b. **Step 2:** Goal (Profit) = \$645                      Price for small mug = \$12 per mug  
                  Price for large mug = \$14 per mug                      Rental fee (Expenses) = \$75

We now have two quantities to solve for but we also know a relationship between the quantities.

$$\text{Number of large mugs} = 2 \times \text{Number of small mugs}$$

We will let  $x$  represent the number of small mugs.

That means  $2x$  will represent the number of large mugs.

**Step 3:** Profit = Revenue from small mugs + Revenue from large mugs - Expenses

$$\text{Profit} = (\text{Price} \times \text{Number of small mugs}) + (\text{Price} \times \text{Number of large mugs}) - \text{Expenses}$$

**Step 4:**  $\$645 = \$12x + \$14(2x) - \$75$

**Step 5:** Solve the equation.

$$\$645 = \$12x + \$14(2x) - \$75$$

$$\$645 = \$40x - \$75$$

$$\$720 = \$40x$$

$$x = 18$$

Elsie needs to sell 18 of the small mugs and 36 ( $2 \times 18$ ) of the large mugs to reach her goal of making \$645.

Check:  $(\$12 \times 18 + \$14 \times 36 - \$75 = \$645)$



**TIP****Avoid Fractions in Equations**

You will avoid fractions in your equations, as seen in **Example 2.5A** part (b), if you let the variable  $x$  represent the smallest quantity in a relationship with two or more unknown quantities.

**EXAMPLE 2.5B****A PROBLEM USING PERCENTS**

A retailer reduced his prices by 15% for a fall sale. What was the regular price of an item on sale at \$123.25?

**SOLUTION**

**Step 2:** Extract and label the data. Identify the unknown and define its symbol.

$$\text{Discount rate} = 15\% \quad \text{Sale price} = \$123.25$$

Let  $P$  represent the regular price.

**Step 3:** Create a word equation that relates the given data to the unknown quantity.

$$\begin{aligned} \text{Sale price} &= \text{Regular price} - \text{Price reduction} \\ &= \text{Regular price} - (\text{Discount rate} \times \text{Regular price}) \end{aligned}$$

**Step 4:** Convert the word equation to an algebraic equation.

$$\$123.25 = P - 0.15P$$

**Step 5:** Solve the equation.

$$\begin{aligned} \$123.25 &= 0.85P \\ P &= \frac{\$123.25}{0.85} = \$145.00 \end{aligned}$$

The original price of the item was \$145.00.

Check:  $(\$145 - 0.15 \times \$145 = \$145 - \$21.75 = \$123.25)$

**EXAMPLE 2.5C****USING TABLES TO ORGANIZE INFORMATION IN A MIXTURES PROBLEM**

Biko Confectionery intends to prepare and sell a Premier Nut Mix consisting of almonds and cashews. The manager obtains almonds at a wholesale cost of \$10.38 per kilogram and cashews at a cost of \$15.50 per kilogram. He wants to mix the nuts in a proportion that produces 12 kg of Premier Mix having an effective wholesale cost no greater than \$12.30 per kilogram. What is the maximum weight of cashews that can be put in the mix?

**SOLUTION**

**Step 2:** The problem here is to determine the **weight** of cashews in a given mixture, so we will let  $C$  represent the weight of cashews in a 12-kg batch. Since the mixture contains only the two types of nuts, the remainder of the 12-kg mixture will be made up of almonds. This gives us  $(12 - C)$  to represent the weight of the almonds.

This problem describes data for different **types of nuts** in a mixture and the **cost** of each type, so these will be our column headings for our table. Since we are looking for the **weight** of cashews used in the mixture, that will also need to be included in the table.



Type of nut	Cost per kg	Weight (kg)	Total cost
Cashews	\$15.50	$C$	$15.50C$
Almonds	\$10.38	$12 - C$	$10.38(12 - C)$
Premier Mix	\$12.30 (maximum)	12	$12.30 \times 12$

**Step 3:** The maximum cost of the mix will be reached when the maximum proportion of the more expensive component (cashews) is used.

$$\text{Total cost of Premier Mix} = \text{Total cost of cashews} + \text{Total cost of almonds}$$

where each total cost is the product of the number of kilograms and the cost per kilogram.

**Step 4:**  $12.30(12) = 15.50C + 10.38(12 - C)$

**Step 5:**  $147.60 = 15.50C + 124.56 - 10.38C$

$$147.60 - 124.56 = 15.50C - 10.38C$$

$$23.04 = 5.12C$$

$$C = 4.50 \text{ kg}$$

The maximum weight of cashews in the batch of Premier Nut Mix is 4.5 kg.

### EXAMPLE 2.5D USING TABLES TO ORGANIZE INFORMATION IN AN ALLOCATION PROBLEM

Digitech Inc. has a stock option incentive program for its employees. In any allocation of options, each of 8 senior managers receives twice as many options as each of 31 middle managers. Each middle manager receives 2.5 times as many options as each of 348 regular employees. The board of directors has just approved the issue of a total of 500,000 stock options. Rounded to the nearest whole number, how many options will each senior manager, middle manager, and regular employee receive?

#### SOLUTION

**Step 2:** The information presented in this problem talks about **types of managers**, **number** of each type of manager, and stock **options** for each type. These will become the headings for our table.

Since middle and senior managers receive more options than regular employees, we will let  $E$  represent the number of options each regular employee will receive. This will avoid the use of fractions in our solution.

Position	Number of individuals	Options per individual	Total options
Regular employee	348	$E$	$348E$
Middle manager	31	$2.5E$	$31(2.5E)$
Senior manager	8	$5E$	$8(5E)$

$$\text{Total number of options} = 500,000$$

**Step 3:** Total number of options = (Number given to regular employees)  
+ (Number given to middle managers)  
+ (Number given to senior managers)

**Step 4:**  $500,000 = 348E + 31(2.5E) + 8(5E)$

**Step 5:**  $500,000 = 348E + 77.5E + 40E$   
 $= 465.5E$   
 $E = \frac{500,000}{465.5} = 1074.11$

Each regular employee will receive 1074 options, each middle manager will receive  $2.5(1074.11) = 2685$  options, and each senior manager will receive  $5(1074.11) = 5371$  options.



**TIP****Look for the Bottom Line**

A good strategy to use when you are overwhelmed with information in a word problem is to look for the “bottom line” question in the problem. In most cases this is the sentence with the question mark. This will give you direction in what the variable should represent and what type of relationship exists between the variables. Next, try to categorize what each sentence is talking about. These categories should be part of the relationship you need to solve the problem.

**EXERCISE 2.5**

Answers to the odd-numbered problems are at the end of the book.

**CALCULATOR-FREE PROBLEMS**

- a. Four buses were chartered to take 177 business students on a field trip. If 9 students travelled by car and the rest filled the buses, how many students were in each bus?
- b. A bookstore customer purchased a binder priced at \$9.50 plus 4 pencils. The total cost of the order was \$12.50 before taxes. How much did each pencil cost?
- c. A computer salesman earns \$400 a week plus a 2% commission on all sales. If he earned \$600 for the week, what was his level of sales?
- d. Samuel won \$2000 in a lottery and gave \$200 to each of his siblings. If he was left with \$600, how many brothers and sisters does Samuel have?
- e. Josh used half of his weekly paycheque to pay his cell phone bill. To earn more money, he sold his skateboard on Kijiji for \$50. How much was his paycheque if he was left with \$170 for the week?

**BASIC PROBLEMS**

1. A Web site had  $\frac{2}{7}$  more hits last month than in the same month of the preceding year. If there were 2655 hits last month, how many were there one year earlier?
2. The retail price of a pair of skis consists of the wholesale cost to the retailer plus the retailer's markup. If skis retailing for \$712 are marked up by 60% of the wholesale cost, what is that wholesale cost?
3. The price tags in Angie's Flower Shop include the 13% Harmonized Sales Tax (HST). How much HST will she report for a plant sold at \$39.55?
4. A stockbroker's commission on a transaction is 2.5% of the first \$5000 of the transaction amount and 1.5% of the remainder. What was the amount of a transaction that generated a total commission of \$227?
5. A caterer has the following price structure for banquets. The first 20 meals are charged the basic price per meal. The next 20 meals are discounted by \$2 each and all additional meals are each reduced by \$3. If the total cost for 73 meals comes to \$1686, what is the basic price per meal?
6. Econocar offers two plans for one-week rentals of a compact car. A rate of \$295 per week includes the first 1000 km. Extra distance costs 15 cents per kilometre. A weekly rate of \$389 allows unlimited driving. Rounded to the nearest kilometre, beyond what driving distance is the unlimited driving plan cheaper?



7. Alicia pays 38% income tax on any additional earnings. She has an opportunity to work overtime at 1.5 times her base wage of \$23.50 per hour. Rounded to the nearest quarter hour, how much overtime must she work to earn enough money (after tax) to buy a canoe that costs \$2750 including sales taxes?

### INTERMEDIATE PROBLEMS

8. Classic Homes has found from experience that there should be 40% as many two-bedroom homes as three-bedroom homes in a subdivision, and twice as many two-bedroom homes as four-bedroom homes. How many homes of each type should Classic build in a new 96-home subdivision?
9. Broadway Mazda usually spends half as much on radio advertising as on newspaper advertising, and 60% as much on television advertising as on radio advertising. If next year's total advertising budget is \$160,000, how much (rounded to the nearest dollar) should be allocated to each form of advertising?
10. A city's commercial construction by-laws require five parking spaces for every 100 square metres of retail rental space in a shopping centre. Four percent of the parking spaces must be large spaces for the physically handicapped. Of the remainder, there must be 40% more regular-size spaces than "small-car" spaces. How many parking spaces of each type are required for a 27,500-square-metre shopping centre?
11. Erin has invested in both an equity mutual fund and a bond mutual fund. Her financial adviser told her that her overall portfolio rose in value by 1.1% last year. Erin noted in the newspaper that the equity fund lost 3.3% last year while the bond fund rose 7.7%. To the nearest 0.1%, what percentage of her portfolio was in the equity fund at the beginning of the year?
12. Steel is an alloy of iron and nickel. A steel recycling company has two piles of scrap steel. Pile A contains steel with 5.25% nickel content; Pile B contains steel with 2.84% nickel. The company has an order for 32.5 tonnes of steel containing 4.15% nickel. How much scrap steel should be taken from each pile for reprocessing?
13. The board of directors of Meditronics Inc. has designated 100,000 stock options for distribution to employees and management of the company. Each of three executives is to receive 2000 more options than each of eight scientists and engineers. Each scientist and engineer is to receive 50% more options than each of 14 technicians. How many options will a person in each position receive?
14. Dash Canada offers two long-distance telephone plans. Plan X costs 6.5 cents per minute for calls between 8 a.m. and 6 p.m. weekdays (business hours) and 4.5 cents per minute at other times. Plan Y costs 5.3 cents per minute any time. Above what percentage of business-hour usage will Plan Y be cheaper?
15. Quality Grocer makes its own bulk "trail mix" by mixing raisins and peanuts. The wholesale cost of raisins is \$3.75 per kg and the cost of peanuts is \$2.89 per kg. To the nearest 0.1 kg, what amounts of peanuts and raisins should be mixed to produce 50 kg of trail mix with an effective wholesale cost of \$3.20 per kg?
16. A firm received a bill from its accountant for \$3310, representing a combined total of 41 billable hours for both the Certified General Accountant (CGA) and her accounting technician for conducting the firm's audit. If the CGA charges her time at \$120 per hour and the technician's time at \$50 per hour, how many hours did each work on the audit?
17. Joan, Stella, and Sue have agreed to form a partnership. For the original capital investment of \$32,760, Sue agrees to contribute 20% more than Joan, and Joan agrees to contribute 20% more than Stella. How much will each contribute?



18. The annual net income of the SGR partnership is to be distributed so that Sven receives 30% less than George, and Robert receives 25% more than George. If the past year's net income was \$88,880, what amount should be allocated to each partner?
19. It takes 20 minutes of machine time to manufacture Product X and 30 minutes of machine time to manufacture Product Y. If the machine operated 47 hours last week to produce a combined total of 120 units of the two products, how many units of Y were manufactured?
20. The tickets for a hockey game cost \$19.00 for the blue section and \$25.50 for the red section. If 4460 tickets were sold for a total of \$93,450, how many seats were sold in each section?
21. Mr. Parker structured his will so that each of his four children will receive half as much from the proceeds of his estate as his wife, and each of 13 grandchildren will receive one-third as much as each child. After his death, \$759,000 remains after expenses and taxes for distribution among his heirs. How much will each child and grandchild receive?
22. To coordinate production in a three-stage manufacturing process, Stage B must be assigned 60% more workers than Stage A. Stage C requires three-quarters as many workers as Stage B. How should the foreman allocate 114 workers among the three stages?
23. Fred has centralized the purchasing and record-keeping functions for his three pharmacies in a single office. The annual costs of the office are allocated to the three stores. The Hillside store is charged \$1000 less than twice the charge to the Barnett store. The Westside store is charged \$2000 more than the Hillside store. What is the charge to the Westside store if the cost of operating the central office for a year is \$27,600?
24. One hundred thousand dollars is to be distributed under a firm's profit-sharing plan. Each of three managers is to receive 20% more than each of 26 production workers. How much will each manager and production worker receive?

## 2.6 The Basic Percentage Problem

In **Section 1.2**, we found a part or portion of a whole amount using percents. We will now look at a more general approach that can be used to compare a portion, or part of a quantity, to the whole amount. One measure of the relative size is the fraction

$$\frac{\text{Portion}}{\text{Base}}$$

where the term *base* is used to represent the whole or entire amount. The fraction is called the *rate*. That is,

$$\text{Rate} = \frac{\text{Portion}}{\text{Base}} \quad (2-1)$$

### THE BASIC PERCENTAGE FORMULA

This relation is also used in a more general way to compare a quantity (the *portion*) to some other standard or benchmark (the *base*). In these cases the portion may be larger than the base. Then the *rate* will be greater than 1 and the percent equivalent rate will be more than 100%.

**L05** Given any two of the three quantities *portion*, *base*, and *rate* you can calculate the unknown quantity by using Formula (2-1). You can choose to do either of the following:

- *First substitute* the known values, and *then rearrange* the equation to solve for the unknown;  
**or**
- *First rearrange* the general formula to isolate the unknown quantity on the left side, and *then substitute* the values for the known quantities on the right side. The former approach helps to keep your focus on a single fundamental relationship.



**TIP****Distinguishing between the Base and the Portion**

The key to solving percentage problems is to distinguish between the base and the portion. The base is always the standard or benchmark to which the portion is being compared.

Ask yourself the following questions to help distinguish between base and portion:

Is this quantity the entire amount that is available?

Is this the quantity I have?

If we look at the answers to these questions in the context of a test mark, say  $\frac{35}{50}$ , then:

The maximum number of marks available, 50, is the base.

The number of marks you achieved, 35, is the portion.

In the wording of problems, the quantity following “of” is almost always the base.

**EXAMPLE 2.6A USING THE BASIC PERCENTAGE FORMULA**

- What is 35.25% of \$215?
- How much is  $0.\overline{66}\%$  of \$2000?
- What percentage is 7.38 kg of 4.39 kg?
- 250% of what amount is \$10?

**SOLUTION**

- a. The question asks us to calculate a part (portion) of a given whole (base). Substituting the known values into Formula (2-1)

$$\text{Rate} = \frac{\text{Portion}}{\text{Base}}$$

we obtain

$$0.3525 = \frac{\text{Portion}}{\$215}$$

Multiply both sides of the equation by \$215 and then switch the left and right sides, giving

$$\text{Portion} = 0.3525 \times \$215 = \$75.79$$

That is, 35.25% of \$215 is \$75.79.

- b. Again, the rate and base are given. As in part (a),

$$\text{Portion} = 0.00\overline{66} \times \$2000 = 0.0066666 \times \$2000 = \$13.33$$

In conclusion, \$13.33 is  $0.\overline{66}\%$  of \$2000.

- c. We are given both the portion and the base for a comparison. Here, 7.38 kg is being compared to the reference amount (base) of 4.39 kg. The answer will be greater than 100% since the portion is larger than the base.

$$\text{Rate} = \frac{\text{Portion}}{\text{Base}} = \frac{7.38}{4.39} = 1.681 = 168.1\%$$

Thus, 7.38 kg is 168.1% of 4.39 kg.

- d. Here, \$10 is 250% of the unknown amount (base). Substituting the known values into

$$\text{Rate} = \frac{\text{Portion}}{\text{Base}}$$

we obtain

$$2.50 = \frac{\$10}{\text{Base}}$$



Hence,

$$2.50(\text{Base}) = \$10$$

and

$$\text{Base} = \frac{\$10}{2.50} = \$4.00$$

Therefore, 250% of \$4.00 is \$10.00.

### EXAMPLE 2.6B SOLVING FOR RATE GIVEN PORTION AND BASE

A battery manufacturer encloses a 50-cent rebate coupon in a package of two AAA batteries retailing for \$4.29. What percent rebate does the coupon represent?

#### SOLUTION

In effect, the question is asking you to compare the rebate to the retail price. Therefore, the retail price is the base in the comparison.

$$\text{Rate} = \frac{\text{Portion}}{\text{Base}} = \frac{\$0.50}{\$4.29} = 0.117 = 11.7\%$$

The manufacturer's percent rebate on the batteries is 11.7%.



#### TIP

#### Units of Portion and Base

The preceding example demonstrates that the portion and base must have the same units when calculating rate. (In other words, both quantities in **Example 2.6B** must be in dollars, or both must be in cents.)

### EXAMPLE 2.6C SOLVING FOR PORTION GIVEN RATE AND BASE

The number of students enrolled in a marketing program represents 18% of the total full-time enrollment in a business college that has 3750 full-time students. How many students are enrolled in other full-time programs at the college?

#### SOLUTION

We can use the rate of enrollment in a marketing program with the total enrollment in the college to find the portion of students in the college enrolled in marketing. That is,

$$\text{Rate} = \frac{\text{Portion}}{\text{Base}}$$

$$0.18 = \frac{\text{Portion}}{3750}$$

$$\text{Portion} = 0.18 \times 3750 = 675 \text{ students in the marketing program.}$$

Therefore, the number of students in other programs is

$$3750 - 675 = 3075 \text{ students.}$$

The business college has 3075 students enrolled in programs other than marketing.



**EXAMPLE 2.6D SOLVING FOR BASE GIVEN PORTION AND RATE**

Juliette sold her 1989 Camry on Kijiji for 32% of what she originally paid for it. If she received \$5500 for the car, what did she pay for it when she bought it new in 1989?

**SOLUTION**

The money she received from the sale of the car is a portion of what she originally paid. Therefore, we can find the base using

$$\begin{aligned} \text{Rate} &= \frac{\text{Portion}}{\text{Base}} \\ 0.32 &= \frac{\$5500}{\text{Base}} \\ 0.32 \times \text{Base} &= \$5500 \\ \text{Base} &= \frac{\$5500}{0.32} = \$17,187.50 \end{aligned}$$

Juliette paid \$17,187.50 for the car when she purchased it new.

**CONCEPT QUESTIONS**

1. What is the percent rate if a quantity is four times the size of the base?
2. What is the percent rate if a quantity is  $\frac{1}{1000}$  of the base?
3. If the percent rate is 1000%, what multiple is the portion of the base?
4. If the percent rate is 0.01%, what fraction is the portion of the base?

**EXERCISE 2.6**

Answers to the odd-numbered problems are at the end of the book.

**BASIC PROBLEMS**

**Calculate dollar amounts accurate to the cent and percent amounts to three-figure accuracy.**

- |   |  |
|---|--|
| 1. Calculate 1.75% of \$350.                    | 2. Calculate $6.\bar{6}\%$ of \$666.66.          |
| 3. What percent is \$1.50 of \$11.50?           | 4. What percent is 88¢ of \$44?                  |
| 5. \$45 is 60% of what amount?                  | 6. \$69 is 30% of what amount?                   |
| 7. What amount is 233.3% of \$75?               | 8. What amount is 0.075% of \$1650?              |
| 9. \$134 is what percent of \$67?               | 10. \$1.34 is what percent of \$655?             |
| 11. 150% of \$60 is what amount?                | 12. $0.58\bar{3}\%$ of \$1500 is what amount?    |
| 13. $7\frac{1}{2}\%$ of what amount is \$1.46?  | 14. $12\frac{3}{4}\%$ of what amount is \$27.50? |
| 15. What percent of \$950 is \$590?             | 16. What percent of \$590 is \$950?              |
| 17. 95% of what amount is \$100?                | 18. $8\frac{1}{3}\%$ of what amount is \$10?     |
| 19. 30 m is what percent of 3 km?               | 20. 500 g is what percent of 2.8 kg?             |
| 21. How much is $\frac{1}{2}\%$ of \$10?        | 22. 0.75% of \$100 is what amount?               |
| 23. \$180 is 120% of what amount?               | 24. \$559.35 is 113% of what amount?             |
| 25. $130\frac{1}{2}\%$ of \$455 is what amount? | 26. 0.0505% of \$50,000 is what amount?          |
| 27. \$281.25 is 225% of what amount?            | 28. 350% of what amount is \$1000?               |
| 29. \$10 is 0.5% of what amount?                | 30. \$1.25 is $\frac{3}{4}\%$ of what amount?    |



31. Cecilia and Nathan estimate their total cost for a vacation in Australia to be \$14,775.
  - a. What percentage is this cost of their combined gross monthly income of \$8775?
  - b. If 72% of their gross monthly income is already consumed by rent, taxes, car payments, and other regular living expenses, what percentage is the trip's cost of their remaining annual disposable income?
32. In a one-month period, a convenience store had sales of \$65,560 from its gas pumps and sales of \$36,740 from other in-store products. What percent of total sales were from gasoline?
33. A 540-mL can of K-9 Diet dog food contains 28% protein, 15.5% fat, and 6% fibre.
  - a. How many millilitres of other ingredients are there in the can?
  - b. The recommended serving for a small dog is  $\frac{5}{8}$  of a can. How many millilitres of protein are in one small-dog serving?
34. A provincial minister of education recently announced that his government's forecast expenditure of \$2.68 billion on education next year represents 23.5% of the provincial budget. Rounded to the nearest million dollars, what is the province's total budget for the next year?
35. Unusually high snowfall during the past winter resulted in Brockton's costs for snow plowing and removal to reach \$320,200. This represents 127% of its budgeted cost. Rounded to the nearest \$100, what amount did Brockton budget for snow clearance?
36. The royalty rate performing artists receive from songs downloaded from Apple iTunes is 5.7%. If a band received royalties from Apple of \$99,736.41 for a year, how many song downloads at \$0.99 each did the band have for that year?
37. Your regular workweek is 7.5 hours per day for five days. If you do not work on seven public holidays and you receive two weeks of vacation, what percentage of the total hours in a year are you actually at work? Assume that a year has exactly 52 weeks.
38. In the month of December, Bernie's Bargain Barn had sales of \$9820 in their clothing department, \$4025 in their shoe department, and \$1830 in accessories. If 17% of the merchandise purchased from the clothing department was returned, 8% from the shoe department was returned, and 3% from accessories was returned, what percent of the total revenue for December were the returns if full refunds were given on all merchandise?
39. Ivory hand soap is advertised as being  $99\frac{44}{100}\%$  pure. (It floats!) How many milligrams of impurities are in a 150-g bar of Ivory soap?
40. An online discount broker charges a transaction fee of \$30 plus an additional 3 cents per share. A full-service broker charges a commission rate of 2.4% of the total dollar value of a stock transaction. Suppose you purchase 200 shares of the Bank of Nova Scotia at \$55.40 per share. What percentage are the total fees charged by the online discount broker of the commission you would pay the full-service broker?

### INTERMEDIATE PROBLEMS

41. A full-service broker charges a commission rate of 2.2% of the total dollar value of a stock transaction. A discount broker charges a transaction fee of \$25 plus an additional five cents per share. Suppose you purchase 800 shares of Talisman Energy at \$21.75 per share. What percentage of the commission fee charged by the full-service broker would you save by using the discount broker?
42. A province's progressive income tax rates are structured as follows: 16% tax on the first \$15,000 of taxable income, 26% on the next \$20,000, 35% on the next \$40,000, and 45% on



any additional taxable income. What percentage is an individual's total income tax of his (taxable) income if his taxable income for a year is:

- a. \$33,000?
- b. \$66,000?
- c. \$99,000?

43. In 2015, Canada's population was 35,750,000 and Japan's population was 126,240,000. Canada's land area is 9,093,500 square kilometres but Japan's area is only 377,835 square kilometres. To the nearest 0.01%, what percentage was Canada's population density (people per square kilometre) of Japan's population density in 2015?
44. A property sold for 250% of what the vendors originally paid for it. What was that original price if the recent selling price was \$210,000?
45. The Calgary Flames hockey team announced that its season ticket sales of 13,020 represents 67.50% of the Scotiabank Saddledome's seating capacity. Rounded to the nearest 100, how many seats were not sold to season ticket holders?
46. Studies have shown that the average adult male requires 7.5 hours of sleep a night and females require 20 minutes more than males. If the average life expectancy in Canada is 82.7 years for women and 78 years for men, what percentage are the male waking hours of a female's waking hours for a lifetime?
47. Stan is a real estate salesperson. He receives 60% of the 4.8% commission that the real estate agency charges on sales. If his income for the past year was \$150,480, what was the dollar value of his sales for the year?
48. A stockbroker is paid 45% of the commission her firm charges her clients. If she personally receives \$134.55 on an \$11,500 transaction, what is the firm's commission rate?
49. A mortality rate indicates the fraction of individuals in a population who are expected to die in the next year.
  - a. If the mortality rate among 35-year-old males is 0.34%, what is the expected number of deaths per year among a province's total of 50,000 such males?
  - b. If 35-year-old males constitute 0.83% of the overall population in a city of 1.45 million, how many deaths of such males are expected in that city in a year?

## 2.7 Percent Change

**L06** When a quantity changes, the amount of the change is often expressed as a percentage of the initial value. That is,

$$\text{Percent change} = \frac{\text{Final value} - \text{Initial value}}{\text{Initial value}} \times 100\%$$

We can write a more compact formula if we define the following symbols:

$$\begin{aligned} V_i &= \text{Initial (or beginning or original or old) value} \\ V_f &= \text{Final (or ending or new) value} \\ c &= \text{Percent change (or its decimal equivalent)} \end{aligned}$$

Then

$$c = \frac{V_f - V_i}{V_i} \times 100\% \quad (2-2)$$

### PERCENT CHANGE

Note that when a quantity doubles, the percent change is 100%. If a quantity triples, the percent change is 200%, and so on.



**EXAMPLE 2.7A** CALCULATING THE PERCENT CHANGE

The minimum wage in Alberta in 2009 was \$8.80 per hour. This increased to \$11.20 per hour in 2015. What is the percent increase in the minimum wage in Alberta for this time period?

**SOLUTION**

From 2009 to 2015,

$$\begin{aligned} c &= \frac{2015 \text{ wage} - 2009 \text{ wage}}{2009 \text{ wage}} \times 100\% = \frac{\$11.20 - \$8.80}{\$8.80} \times 100\% \\ &= \frac{\$2.40}{\$8.80} \times 100\% \\ &= 27.27\% \end{aligned}$$

The minimum wage in Alberta increased by 27.27% from 2009 to 2015.

**TIP****Order Matters, Language Matters**

The order of the final and initial values in the numerator is important—it determines the sign of the percent change. If a quantity decreases in size, its percent change is negative.

To illustrate a point on language, consider the example of a company whose sales declined from \$4 million in Year 1 to \$3 million in Year 2. The percent change in sales is

$$\begin{aligned} \frac{\text{Year 2 sales} - \text{Year 1 sales}}{\text{Year 1 sales}} \times 100\% &= \frac{\$3 \text{ million} - \$4 \text{ million}}{\$4 \text{ million}} \times 100\% \\ &= \frac{-1}{4} \times 100\% \\ &= -25\% \end{aligned}$$

We can say either “the sales changed by  $-25\%$ ” or “the sales decreased by  $25\%$ ” from Year 1 to Year 2. The direction of the change may be indicated either by an algebraic sign or by a descriptive word such as “rose,” “fell,” “increased,” or “decreased.” However, it would be incorrect and potentially confusing to say that “the sales decreased by  $-25\%$ .”

**EXAMPLE 2.7B** CALCULATING THE PERCENT CHANGE

The share price of Klondike Resources rose from \$2 on January 1, 2015 to \$4 on December 31, 2015. It fell back to \$2 by December 31, 2016. Calculate the percent change in share price during 2015, during 2016, and during the entire two-year period.

**SOLUTION**

For 2015,

$$c = \frac{\text{Dec. 31, 2015 price} - \text{Jan. 1, 2015 price}}{\text{Jan. 1, 2015 price}} \times 100\% = \frac{\$4 - \$2}{\$2} \times 100\% = 100\%$$

Similarly, for 2016,

$$c = \frac{\$2 - \$4}{\$4} \times 100\% = -50\%$$

For the entire two years,

$$c = \frac{\$2 - \$2}{\$2} \times 100\% = 0\%$$

The share price increased by 100% in 2015 and then decreased by 50% in 2016. For the entire two-year period, there was no net price change.